INTERFERENCE CHANNELS

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Based on joint work with Chandra Nair (CUHK)

Fourth Van Der Meulen Seminar
IEEE Benelux Information Theory Chapter

Dedication: In honor of Thomas Cover

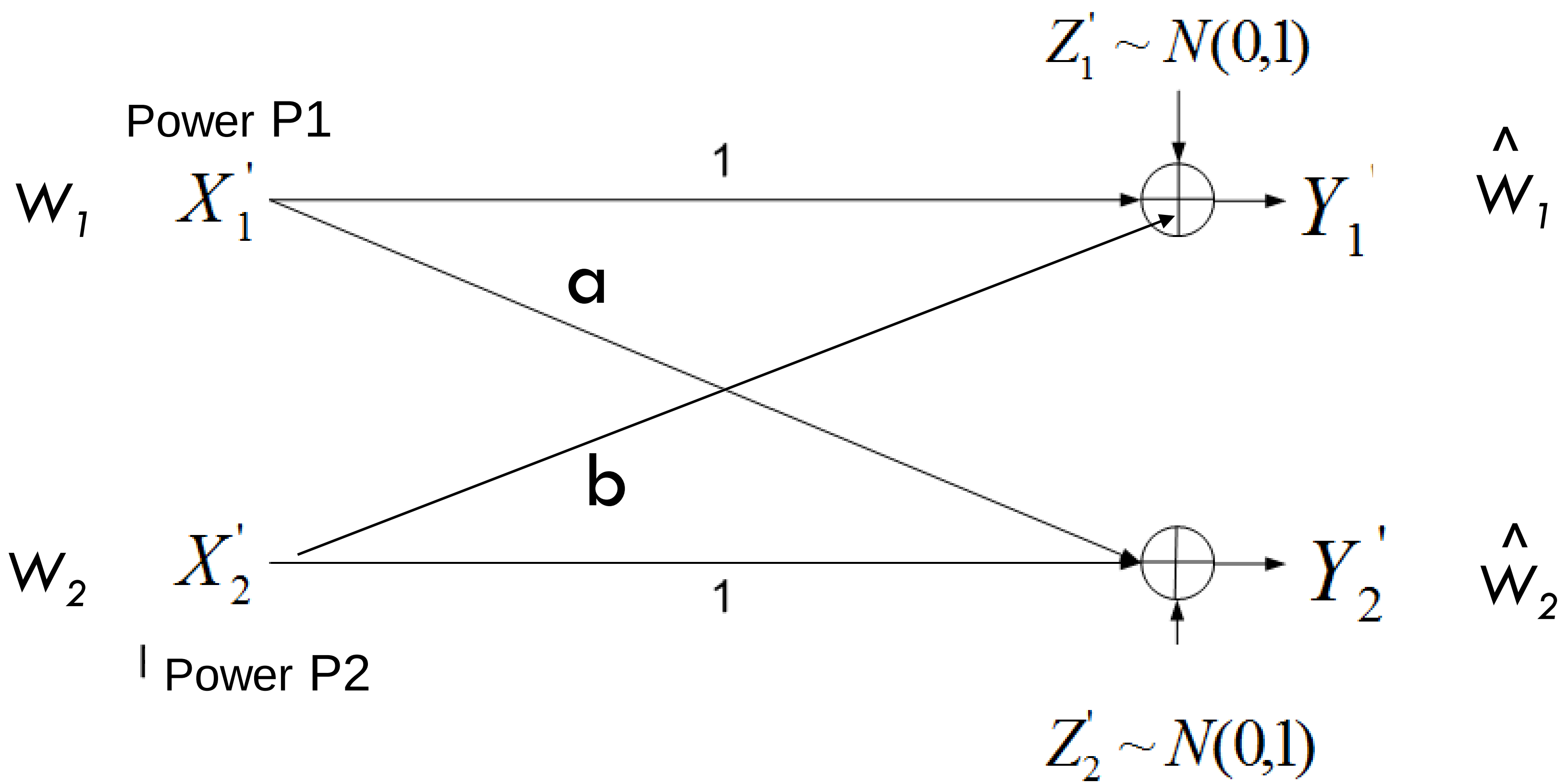
Thomas Cover – 1938-2012

“There are a lot of simple and shocking statements that come out of probability and statistics. It is the existence of these as yet unfound statements that drives my interest in the field.

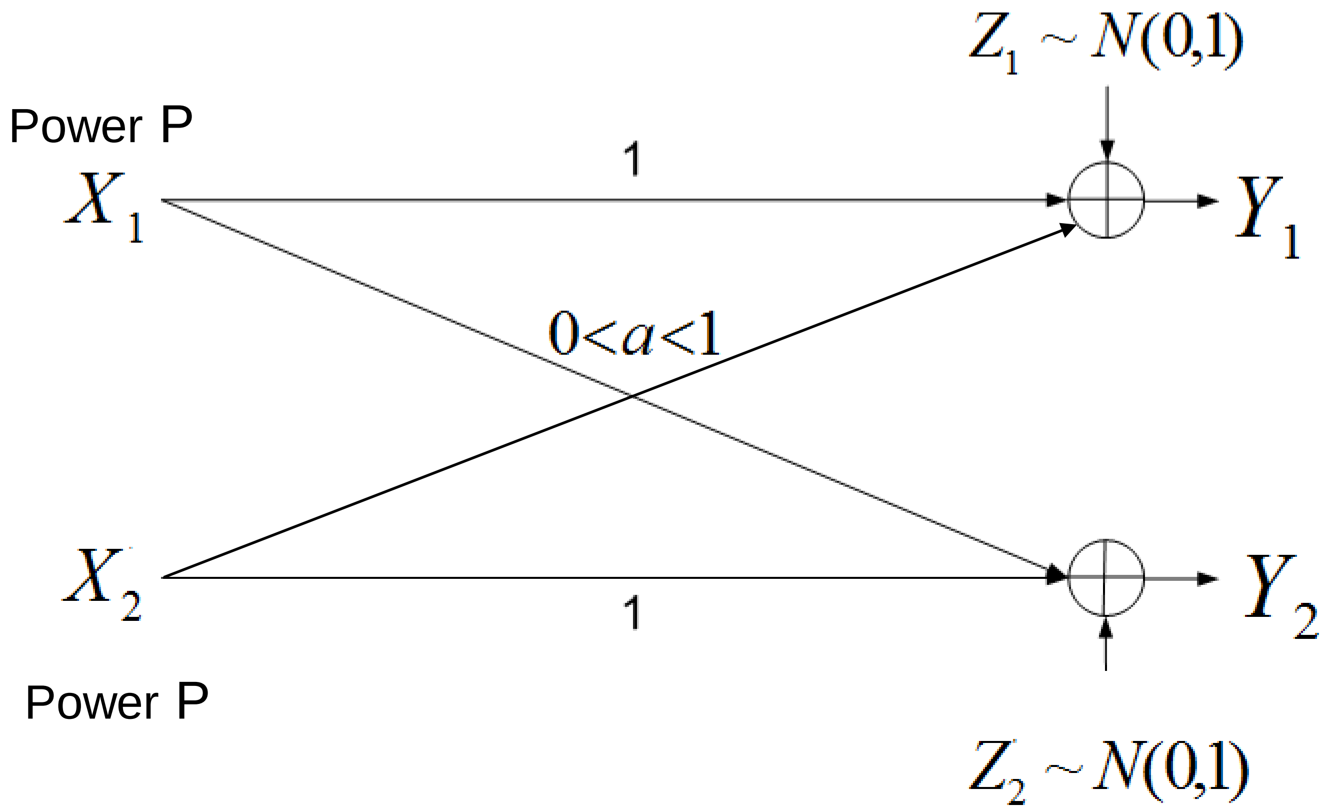
Summary

- Gaussian Interference Channel - standard form
- Brief history
- Z-Interference channel
- Symmetric Interference channel

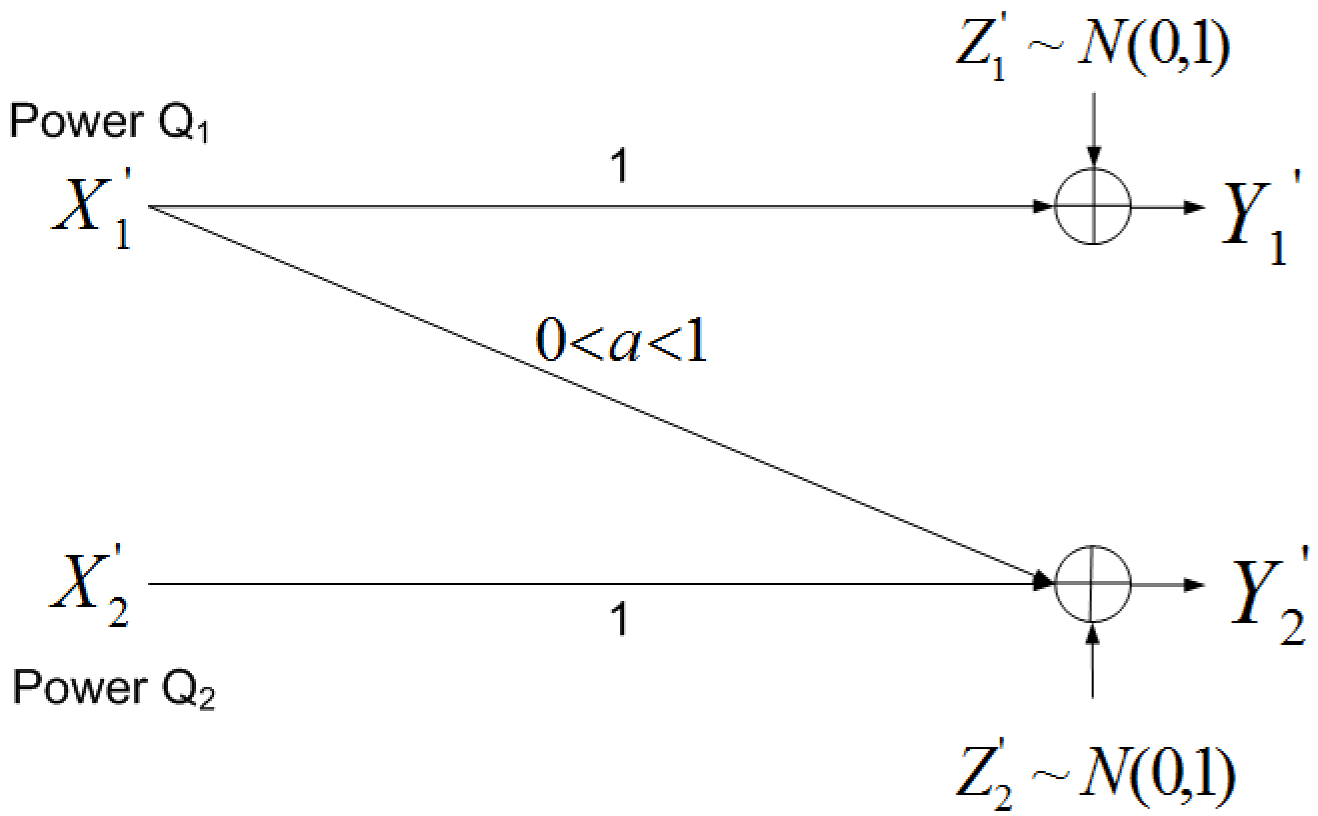
Standard Gaussian Interference Channel



Symmetric Gaussian Interference Channel



Z-Gaussian Interference Channel



The possibilities:

Things that we can do with interference:

1. Ignore (take interference as noise (IAN)
2. Avoid (divide the signal space (TDM/FDM))
3. Partially decode both interfering signals
4. Partially decode one, fully decode the other
5. Fully decode both (only good for strong interference, $a \geq 1$)

Brief history

- Carleial (1975): Very strong interference does not reduce capacity ($a^2 \geq 1+P$)

- Sato (1981), Han and Kobayashi (1981): Strong interference ($a^2 \geq 1$) : IFC behaves like 2 MACs

- Motahari, Khandani (2009), Shang, Kramer and Chen (2009), Annapureddy, Veeravalli (2009):

  Very weak interference ($2a(1+a^2P) \leq 1$) :

  $\Longrightarrow$ Treat interference as noise (IAN)

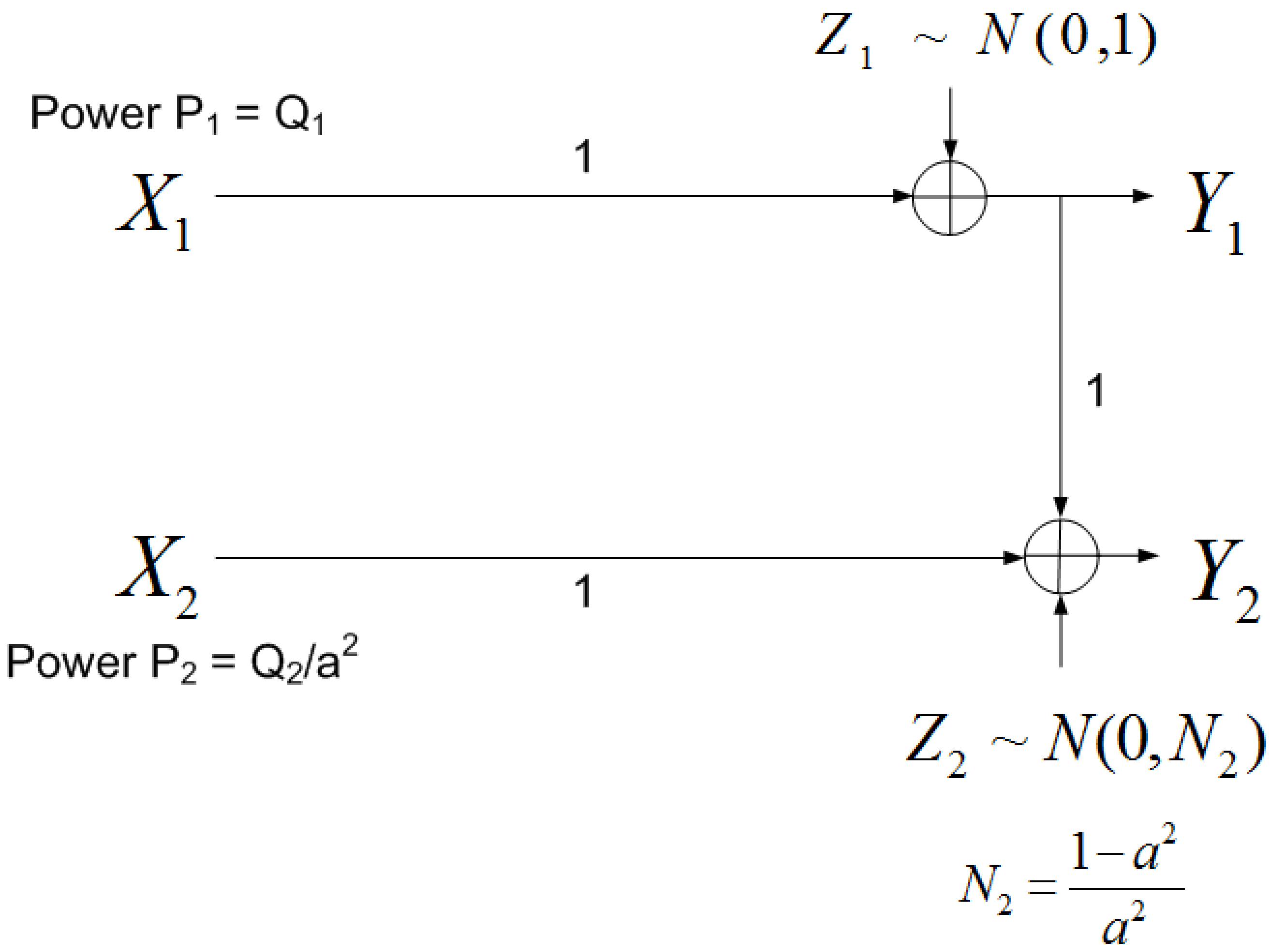
History (continued)

- Sason (2004): Symmetrical superposition to beat TDM – found part of optimal choice for α
- Etkin, Tse, Wang (2008): capacity to within 1 bit, good heuristical choice of $\alpha P=1/a^2$

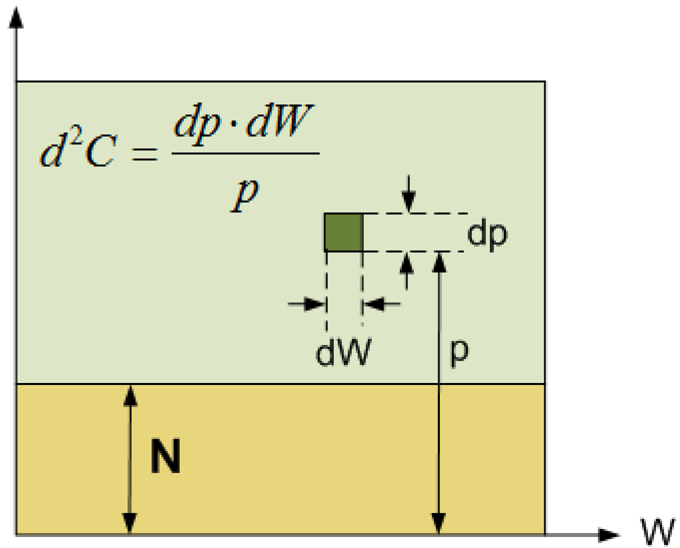
Summary: Z interference Channels

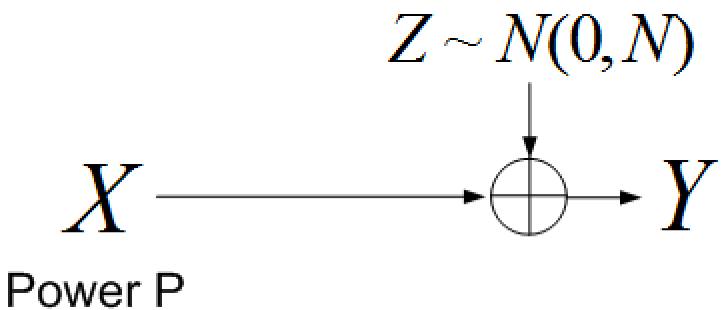
- Z-Gaussian Interference Channel as a degraded interference channel
- Discrete Memoryless Channel as a band limited channel
- Multiplex Region: growing **Noisebergs**
- Overflow Region: back to superposition

Degraded Gaussian Interference Channel



Differential capacity

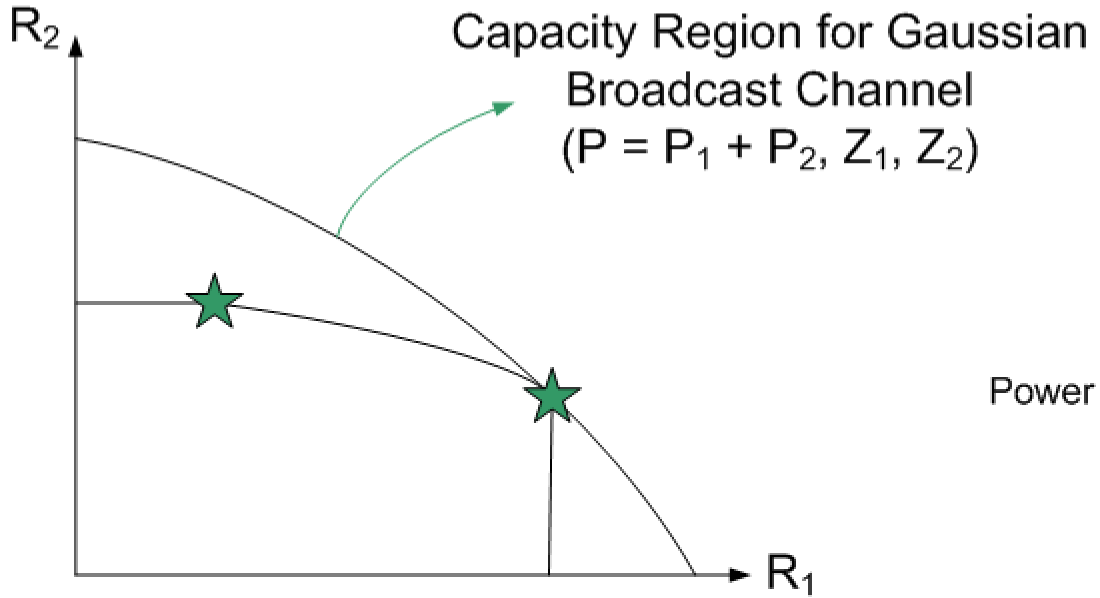


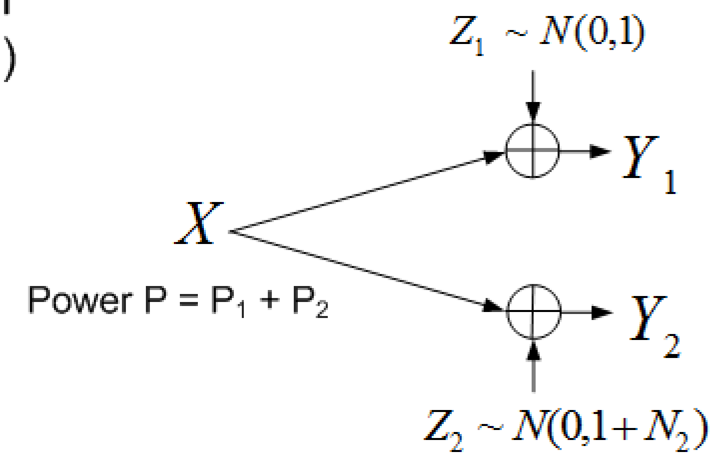


$$C = \iint d^2C = \frac{1}{2}\log\left(1 + \frac{P}{N}\right)$$

Discrete memoryless channel as a band limited channel

Gaussian Broadcast Channel

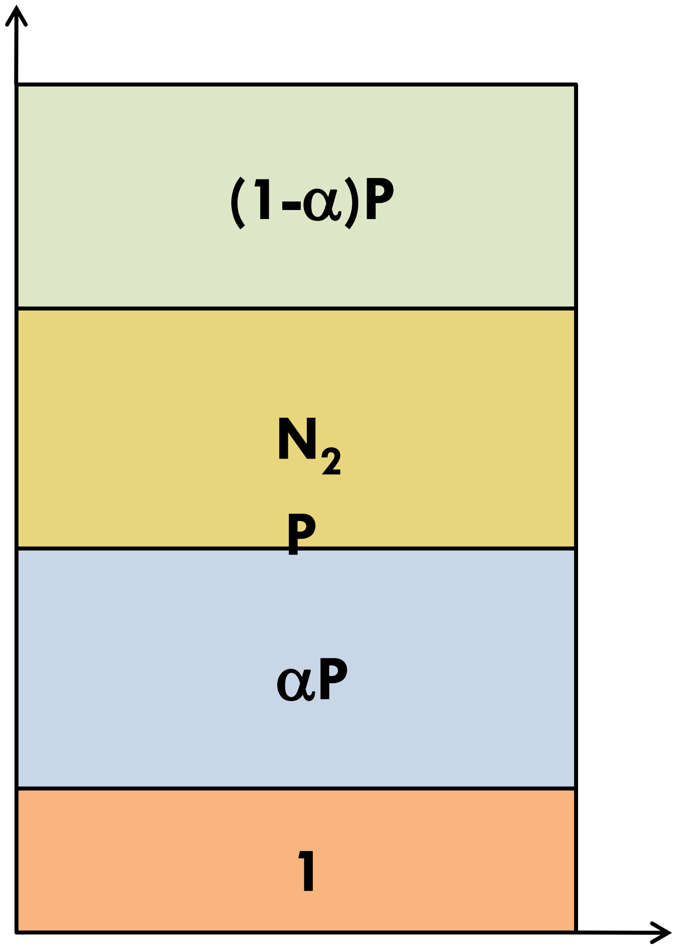


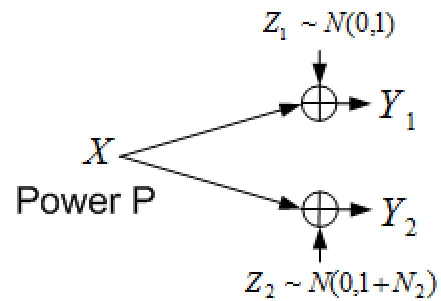


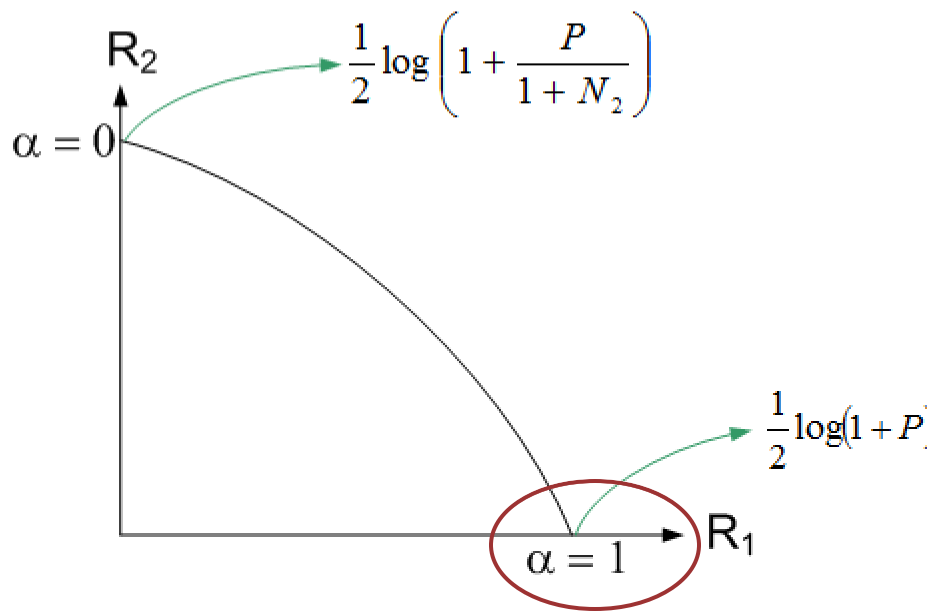
$$C_{BC}\{R_1, R_2\}: \quad 0 \le R_1 \le \frac{1}{2}\log(1 + \alpha P)$$

$$0 \le \alpha \le 1 \quad 0 \le R_2 \le \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P}{1 + N_2 + \alpha P}\right)$$

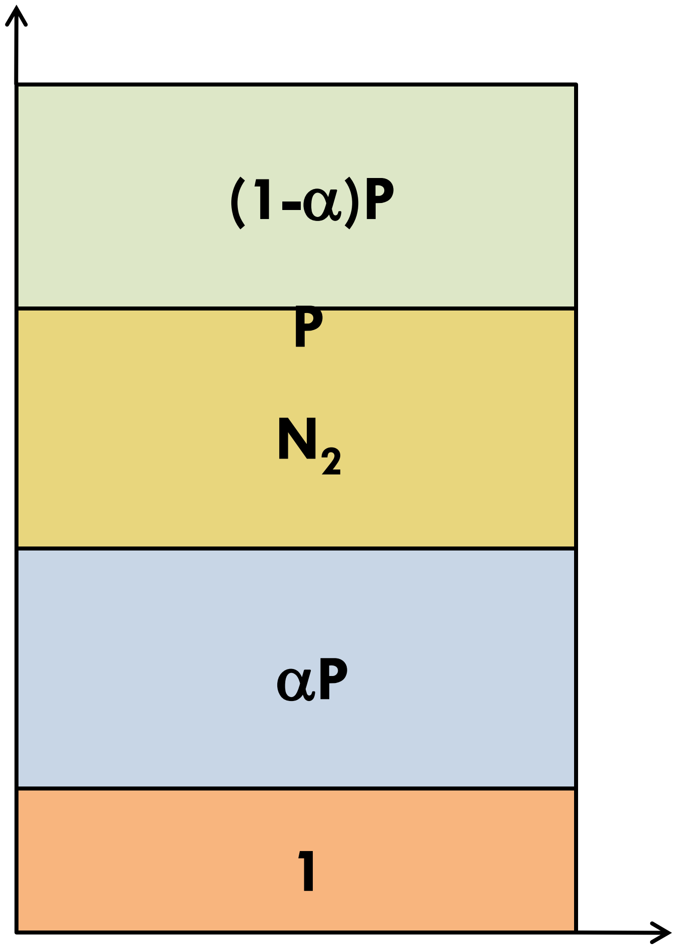
Superposition coding

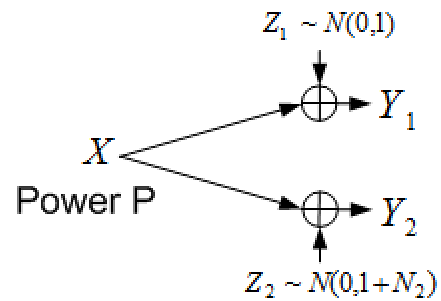


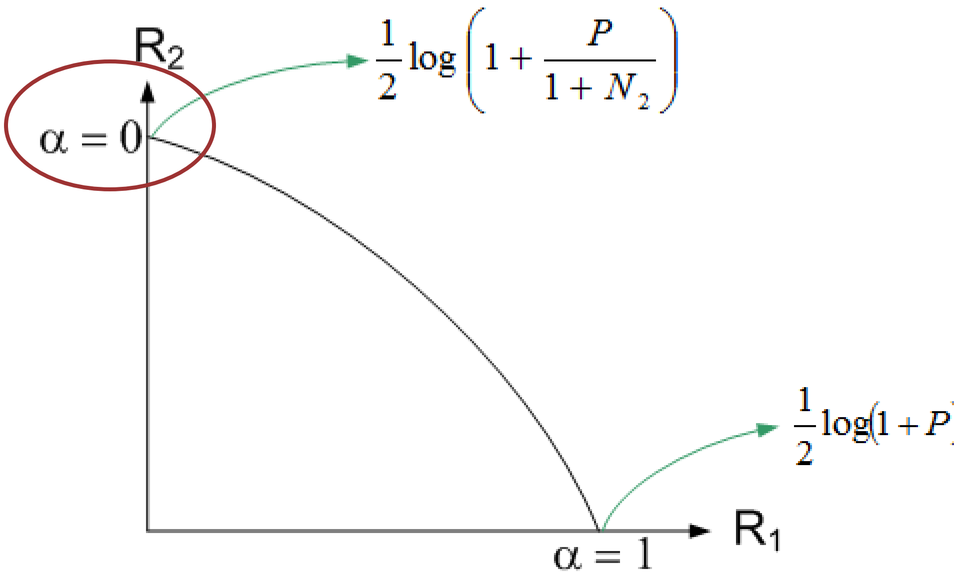




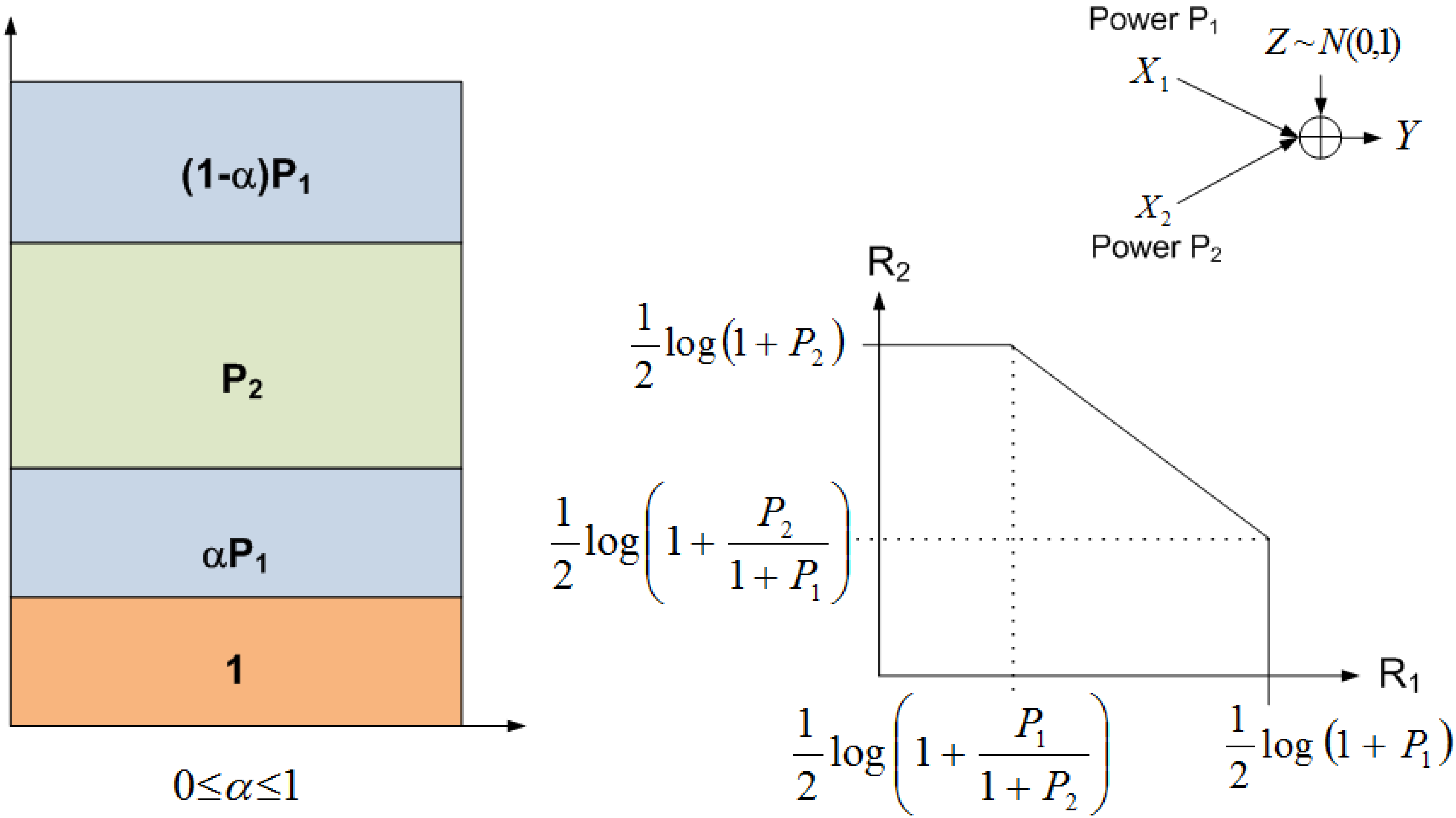
Superposition coding





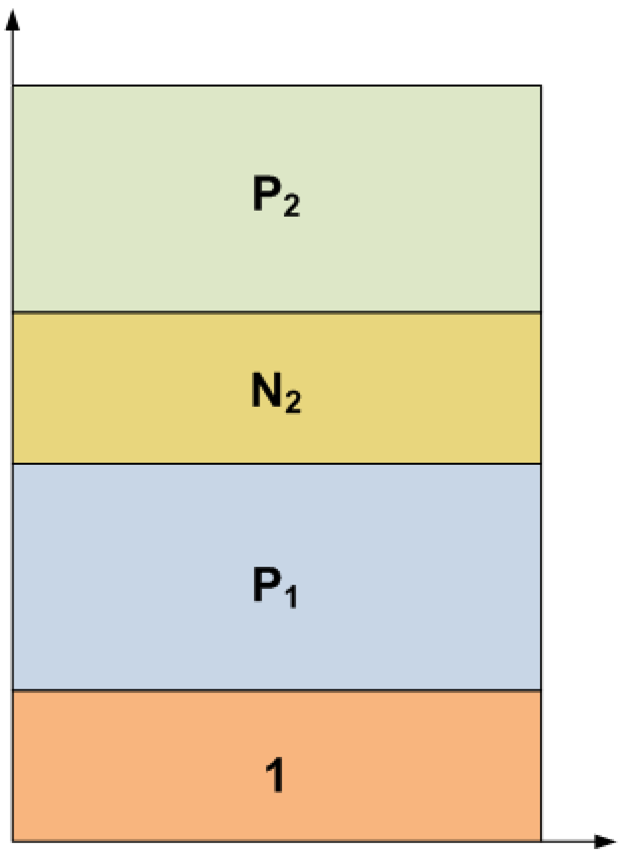


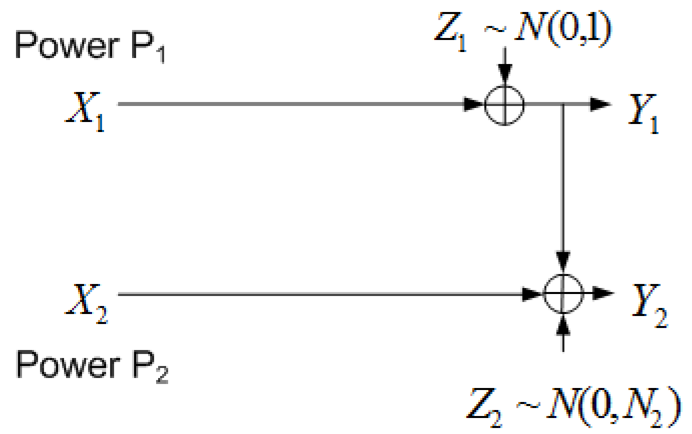
Multiple Access Channel

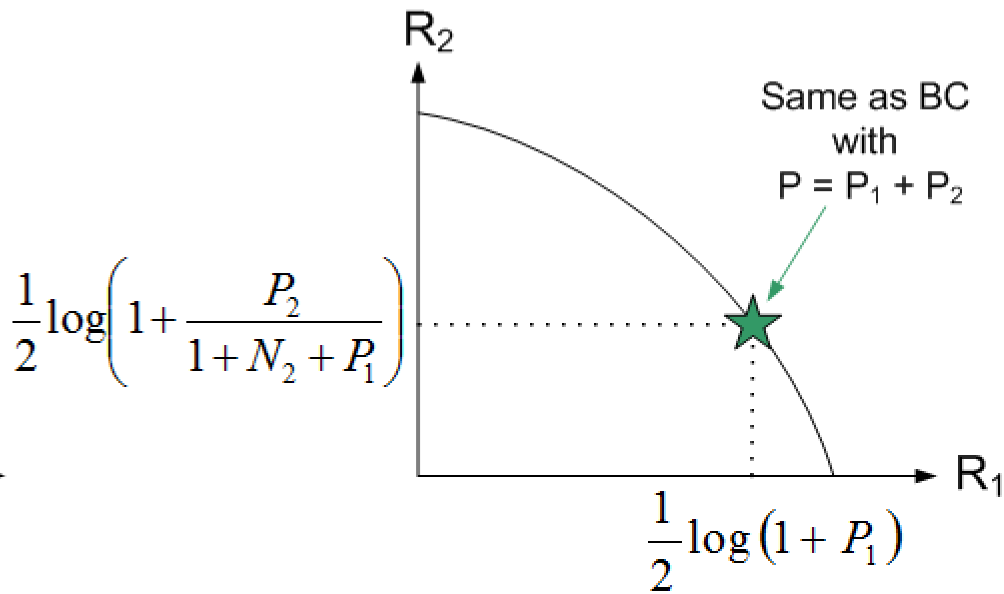


Degraded Interference Channel

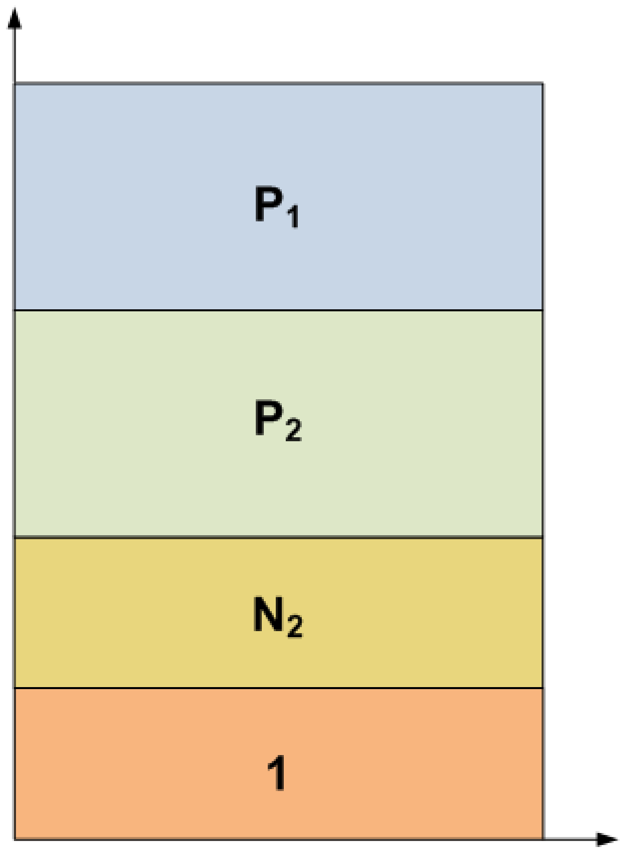
- One Extreme Point

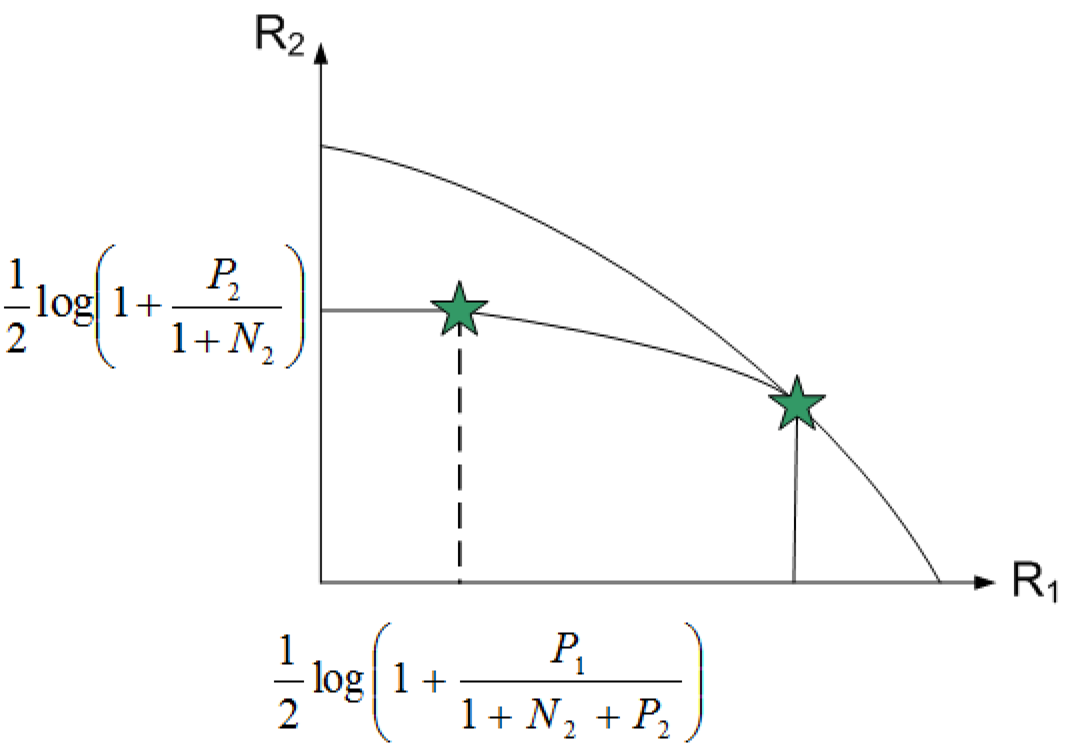




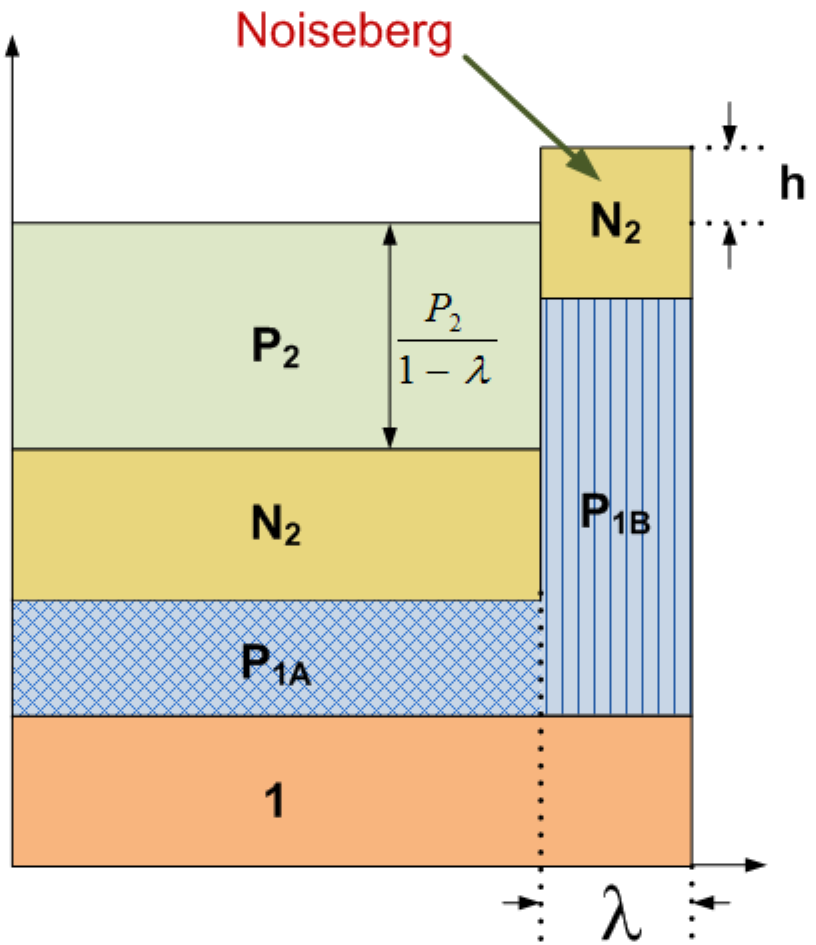


Degraded Interference Channel
- Another Extreme Point

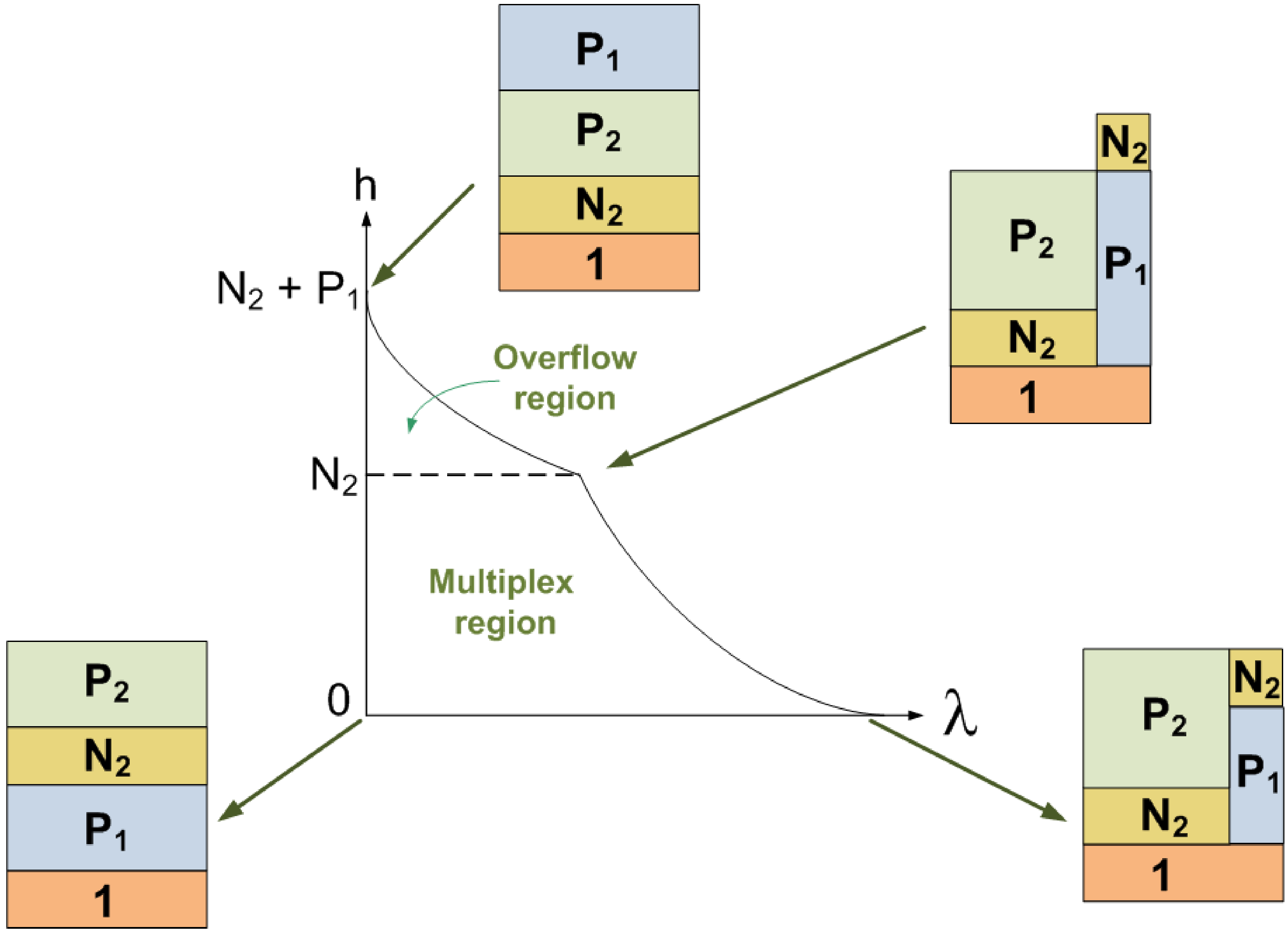




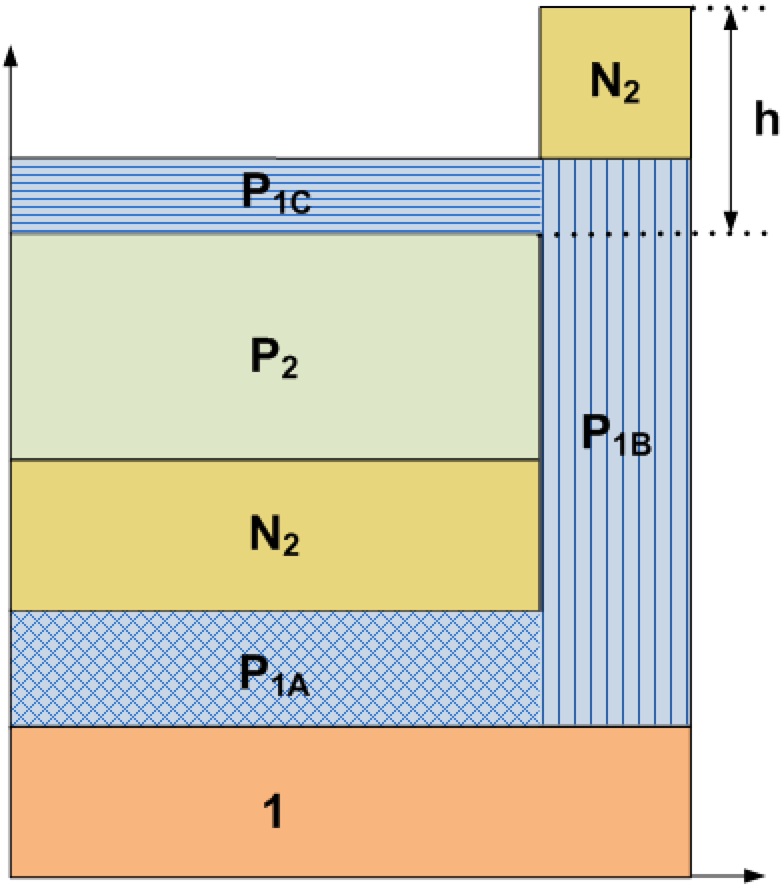
Intermediary Points (Multiplex Region)



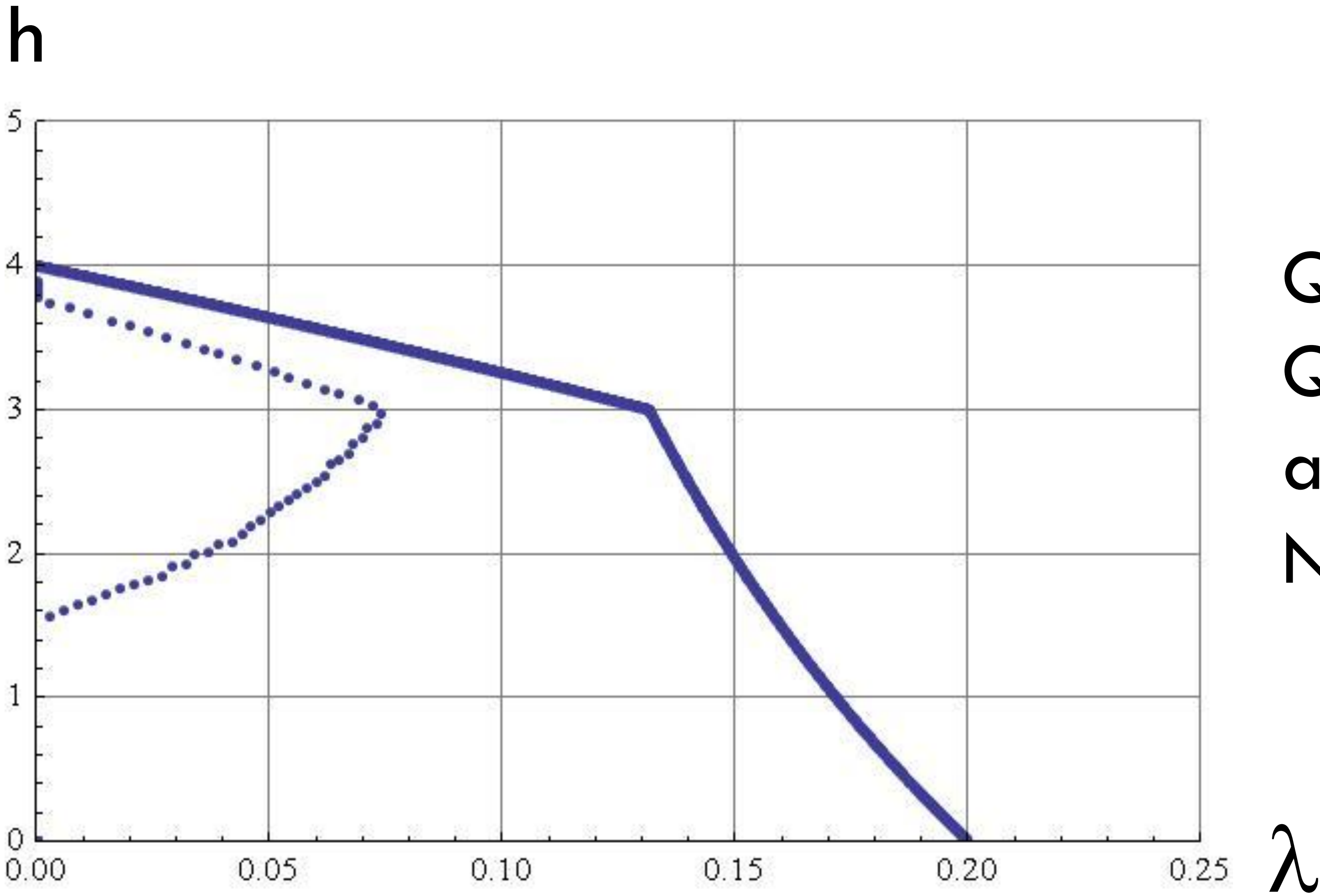
Admissible region for (λ, h)



Intermediary Point (Overflow Region)



Admissible region



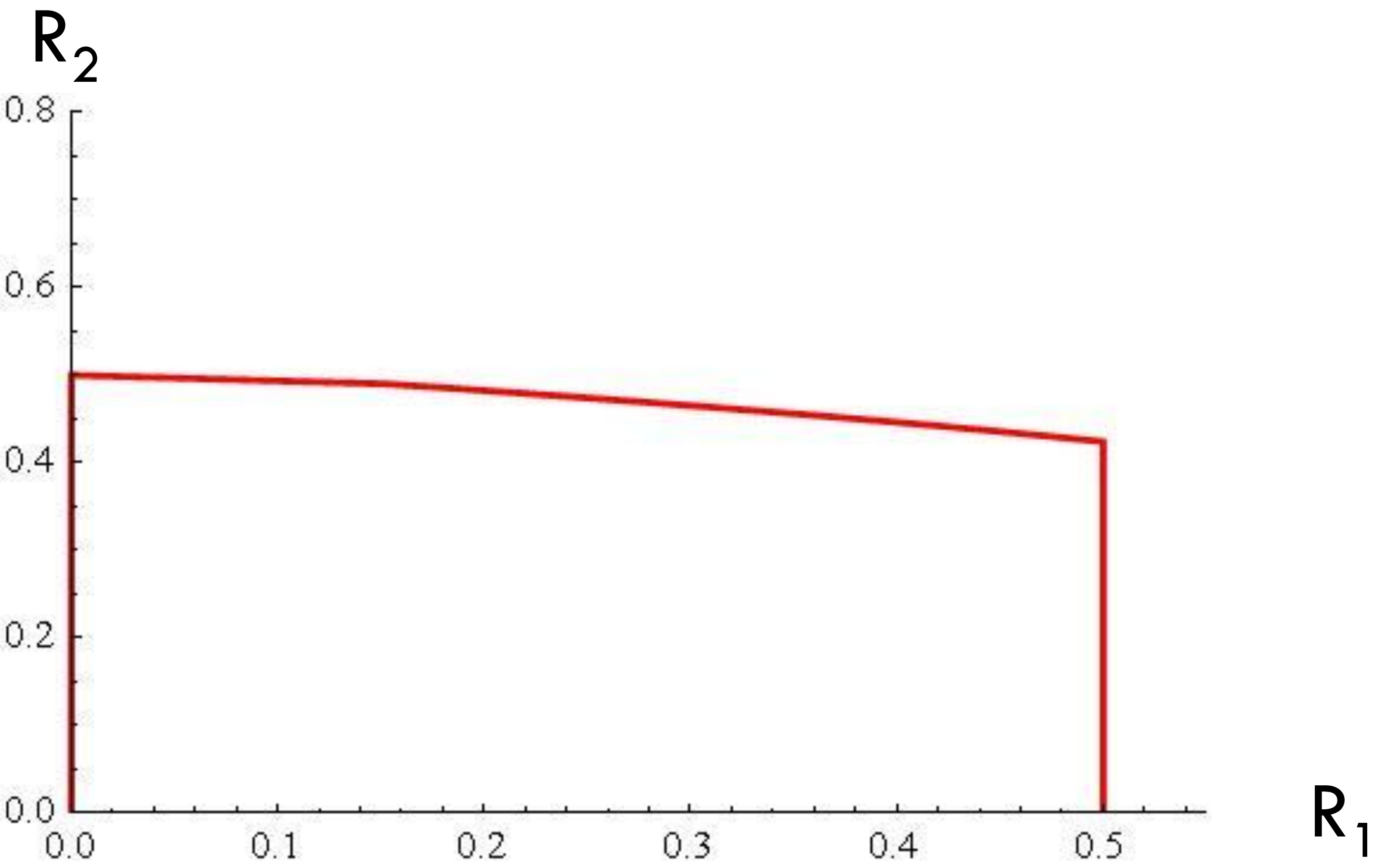
$Q_1 = 1$

$Q_2 = 1$

$a = 0.5$

$N_2 = 3$

The Z-Gaussian Interference Channel Rate Region



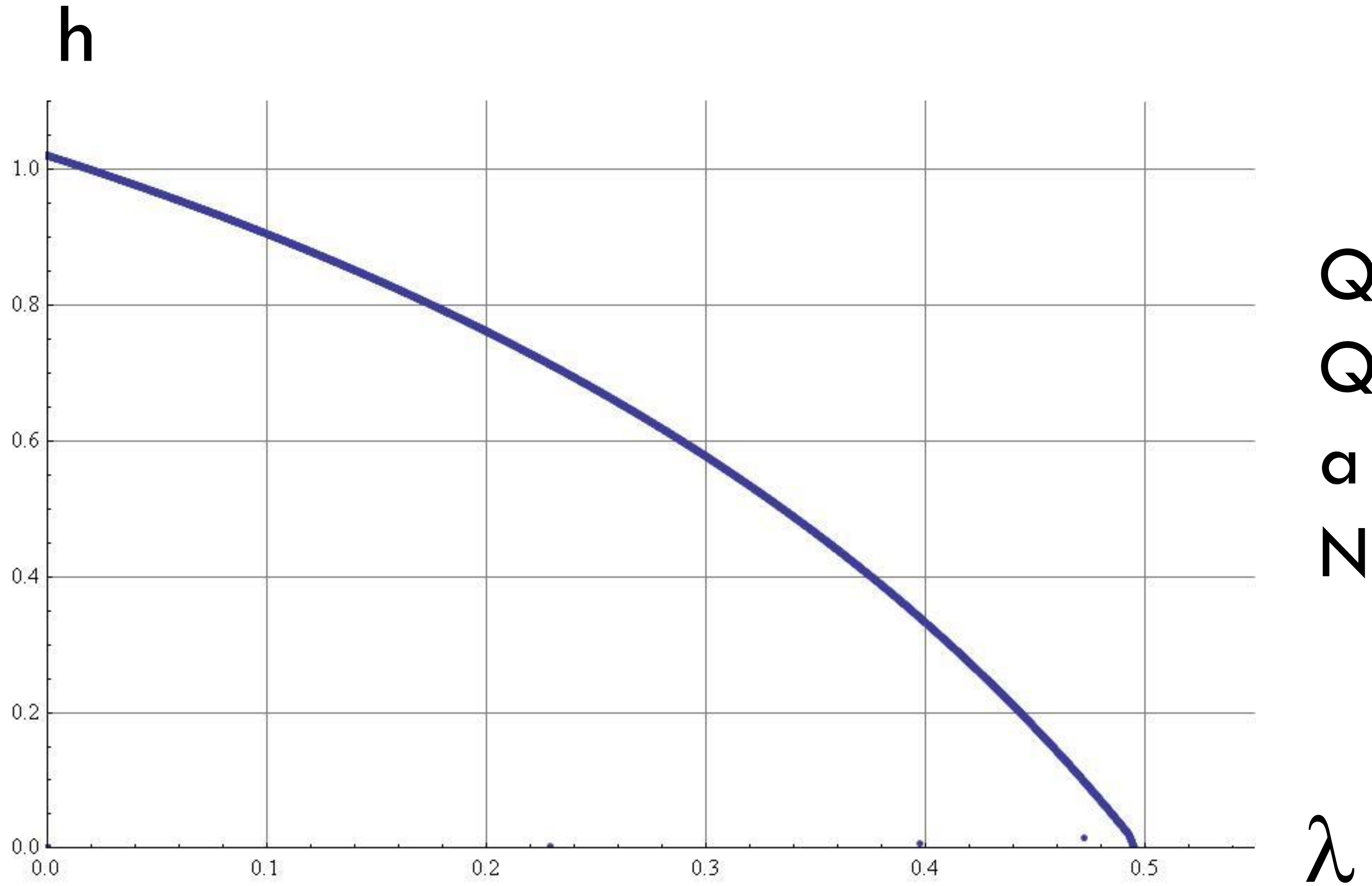
$Q_1 = 1$

$Q_2 = 1$

$a = 0.5$

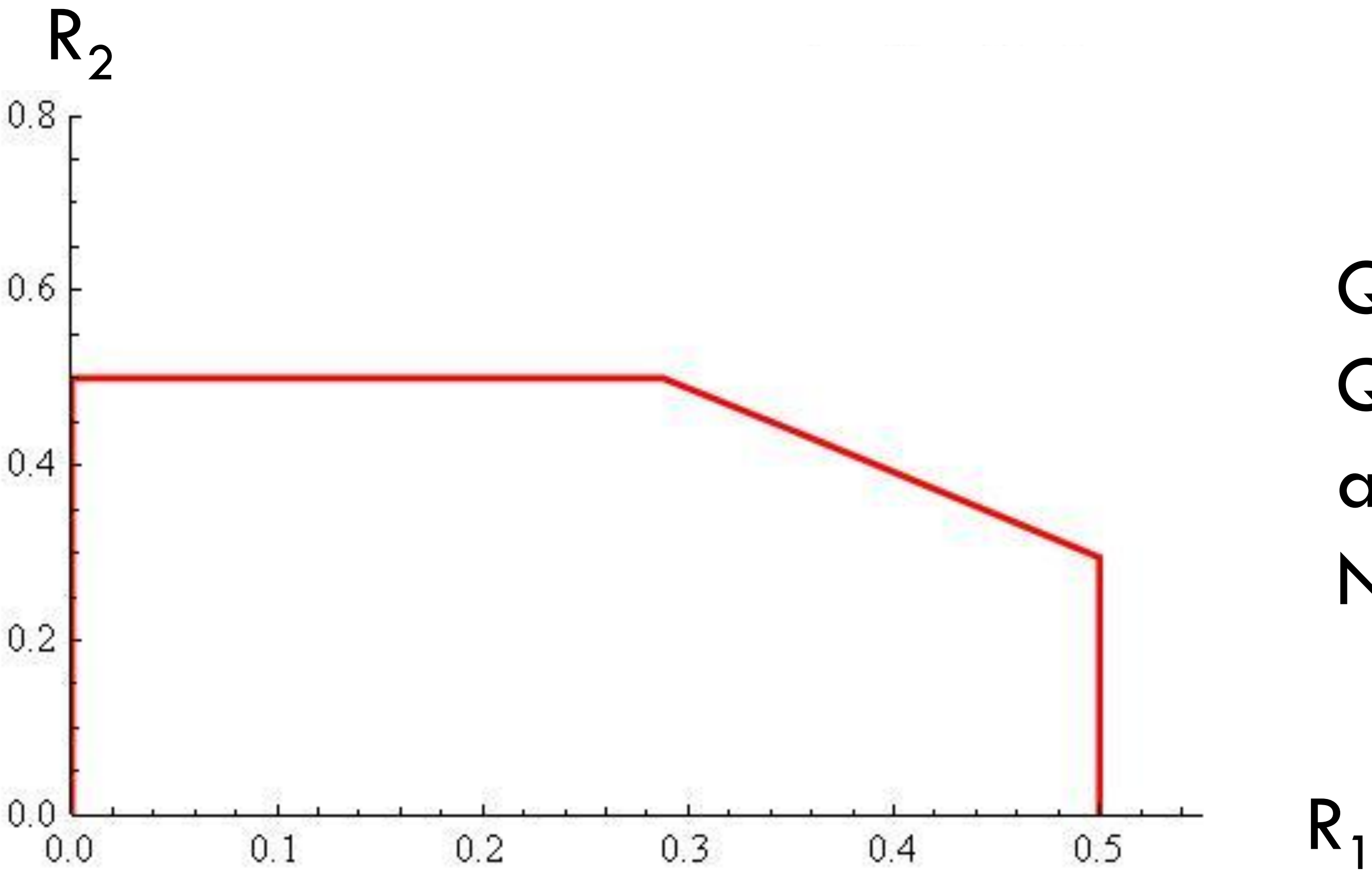
$N_2 = 3$

Admissible region



Q_1=1
Q_2 = 1
a = 0.99
N_2 = 0.02

The Z-Gaussian Interference Channel Rate Region



$Q_1 = 1$

$Q_2 = 1$

$a = 0.99$

$N_2 = 0.02$

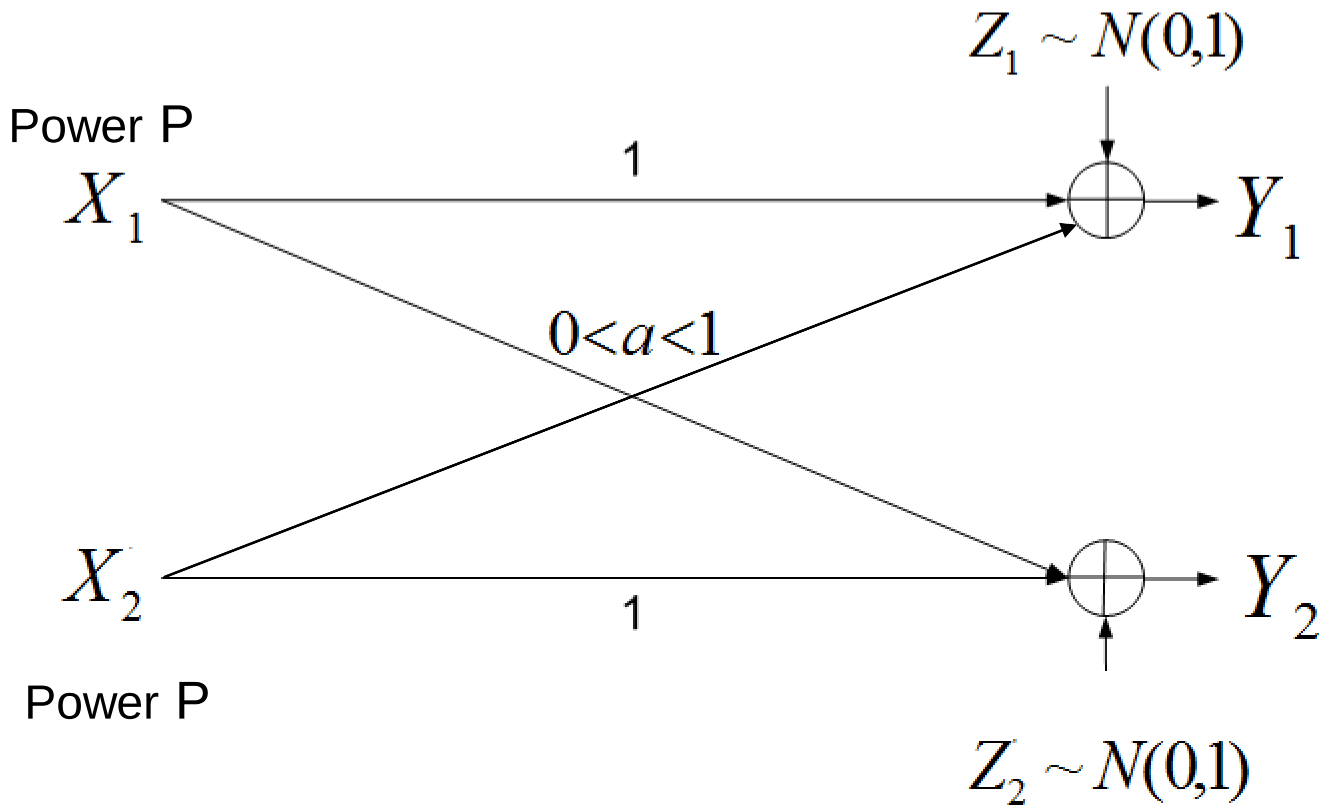
Remarks

- This is Han-Kobayashi region for Gaussian signaling (Zhao et al., ISIT-2012)

- Simple 2-D parameter space: (λ, h)

- Need entropy power-like inequality to establish capacity region

Symmetric Gaussian Interference Channel



Symmetric Interference Channels

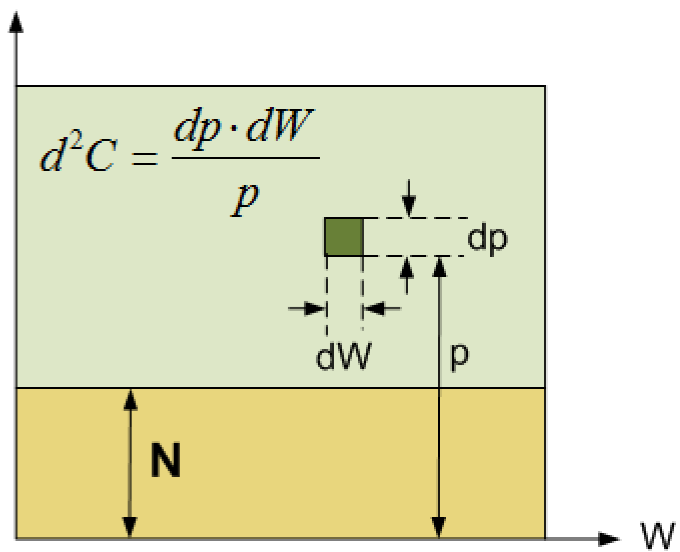
- Discrete time channel seen as a band limited channel – differential capacity

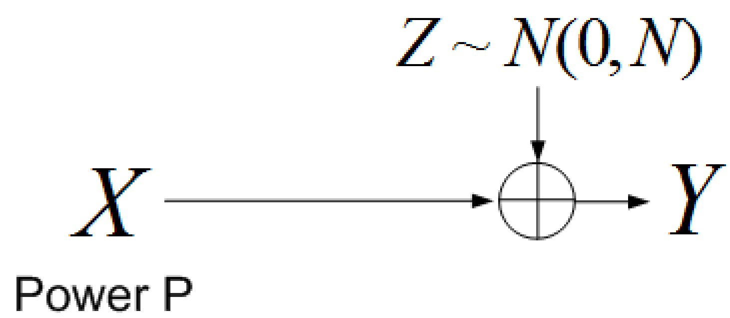
- Concave envelopes

- Symmetric and Asymmetric Superposition

- Phase transitions in parameter space

Differential capacity



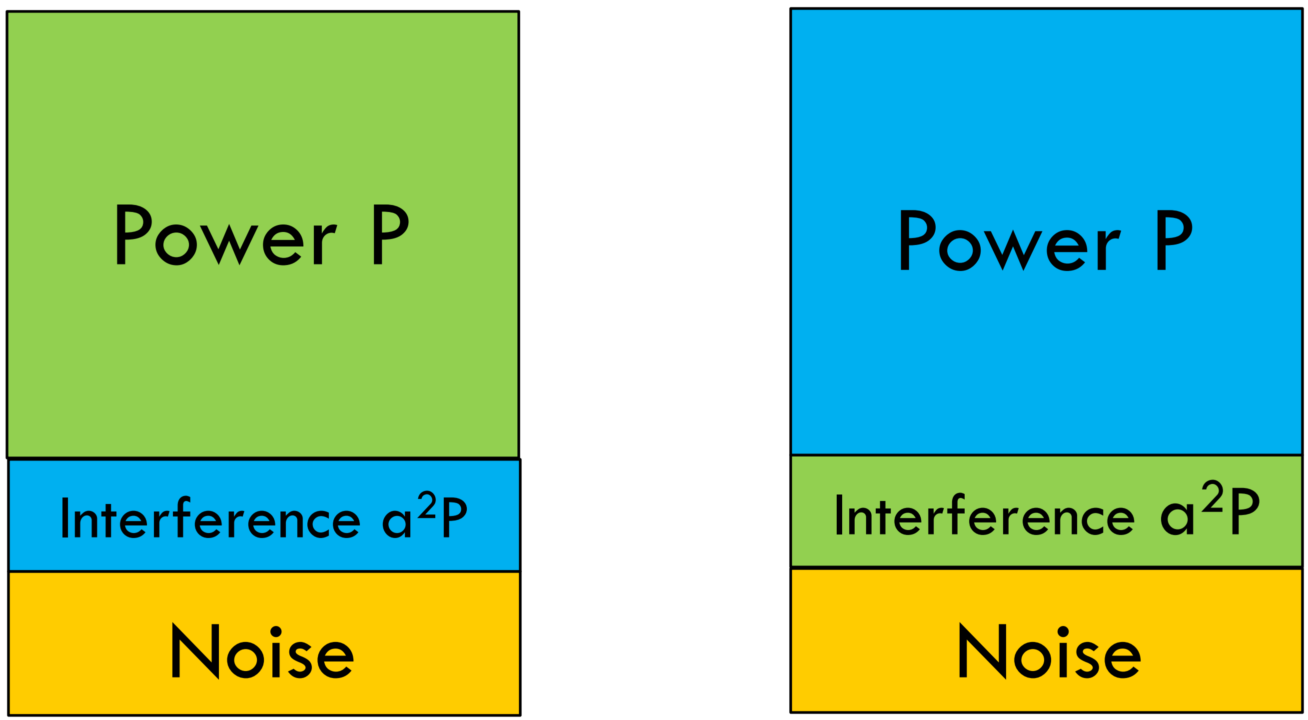


$$C = \iint d^2C = \frac{1}{2}\log\left(1 + \frac{P}{N}\right)$$

Discrete time channel seen as a band limited channel

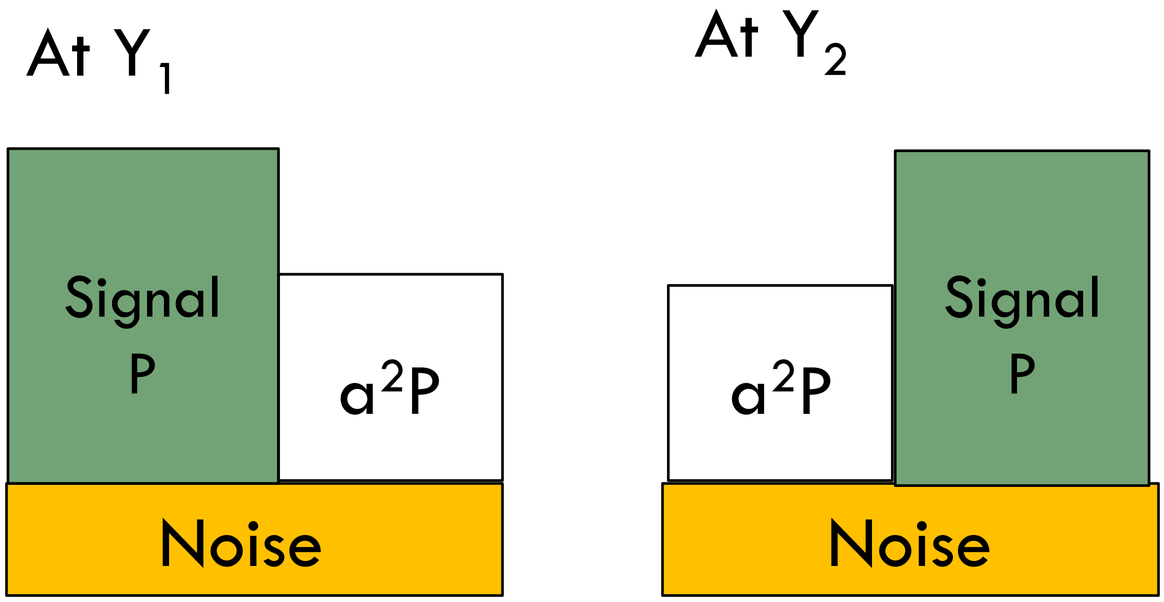
Interference channel: Spectra at Y_1 and Y_2

- At Y_1 At Y_2



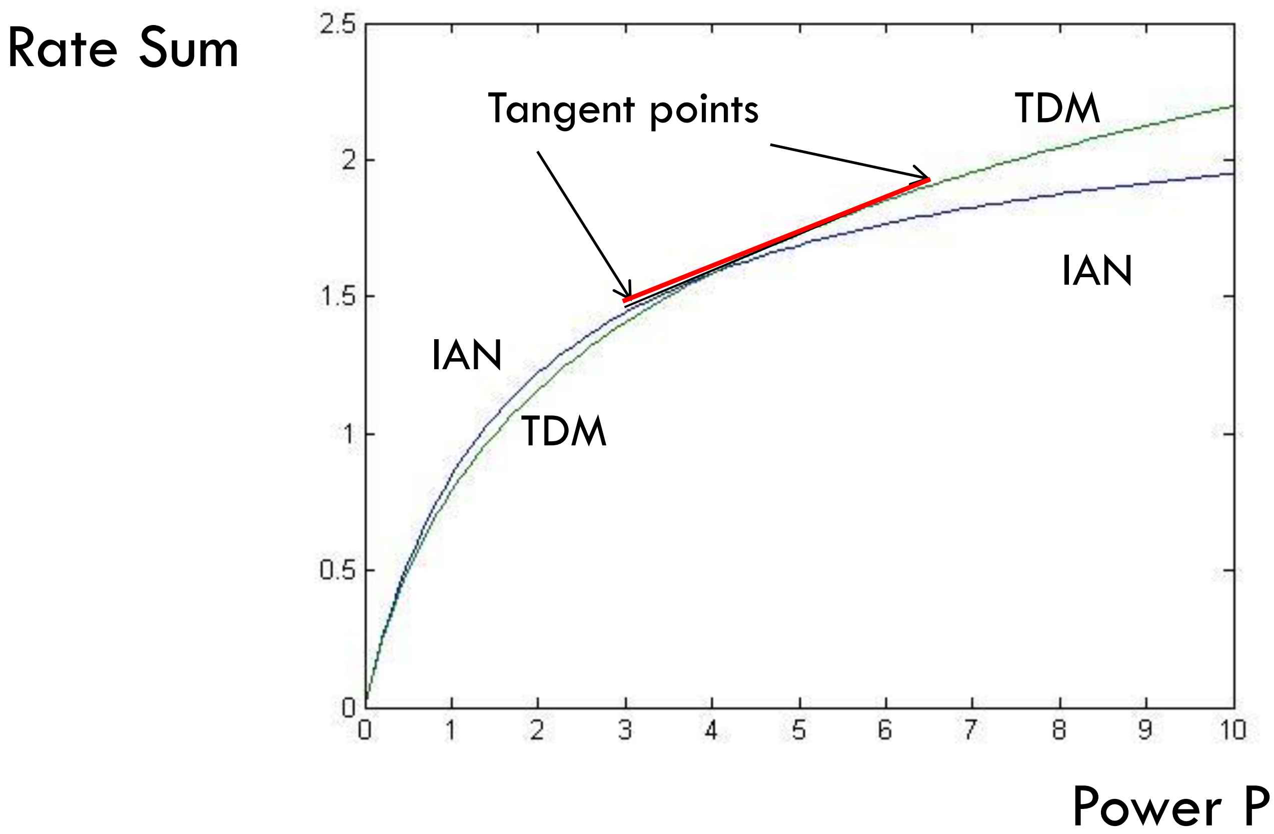
IAN: $R_1 + R_2 \leq \log\left(1 + \frac{P}{1+a^2P}\right)$

Interference Channel: TDM/FDM:



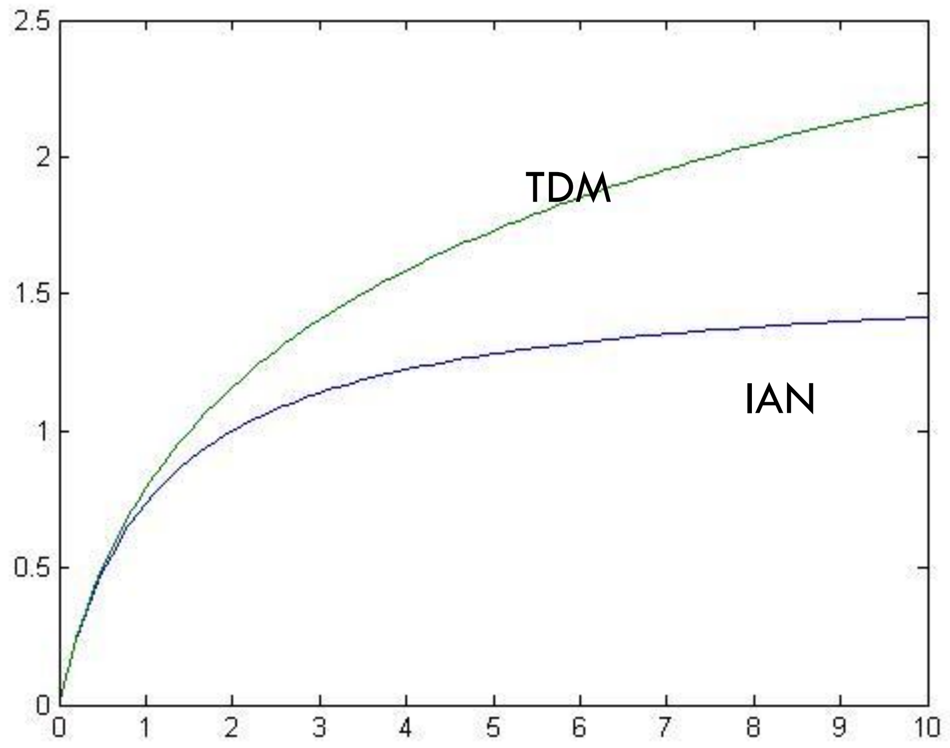
$$R_1 + R_2 \leq \frac{1}{2}\log(1 + 2P)$$

Concave Envelope
IAN vs TDM/FDM, a^2=0.25



Multiplex domination
IAN vs TDM/FDM, $a^2=0.5$

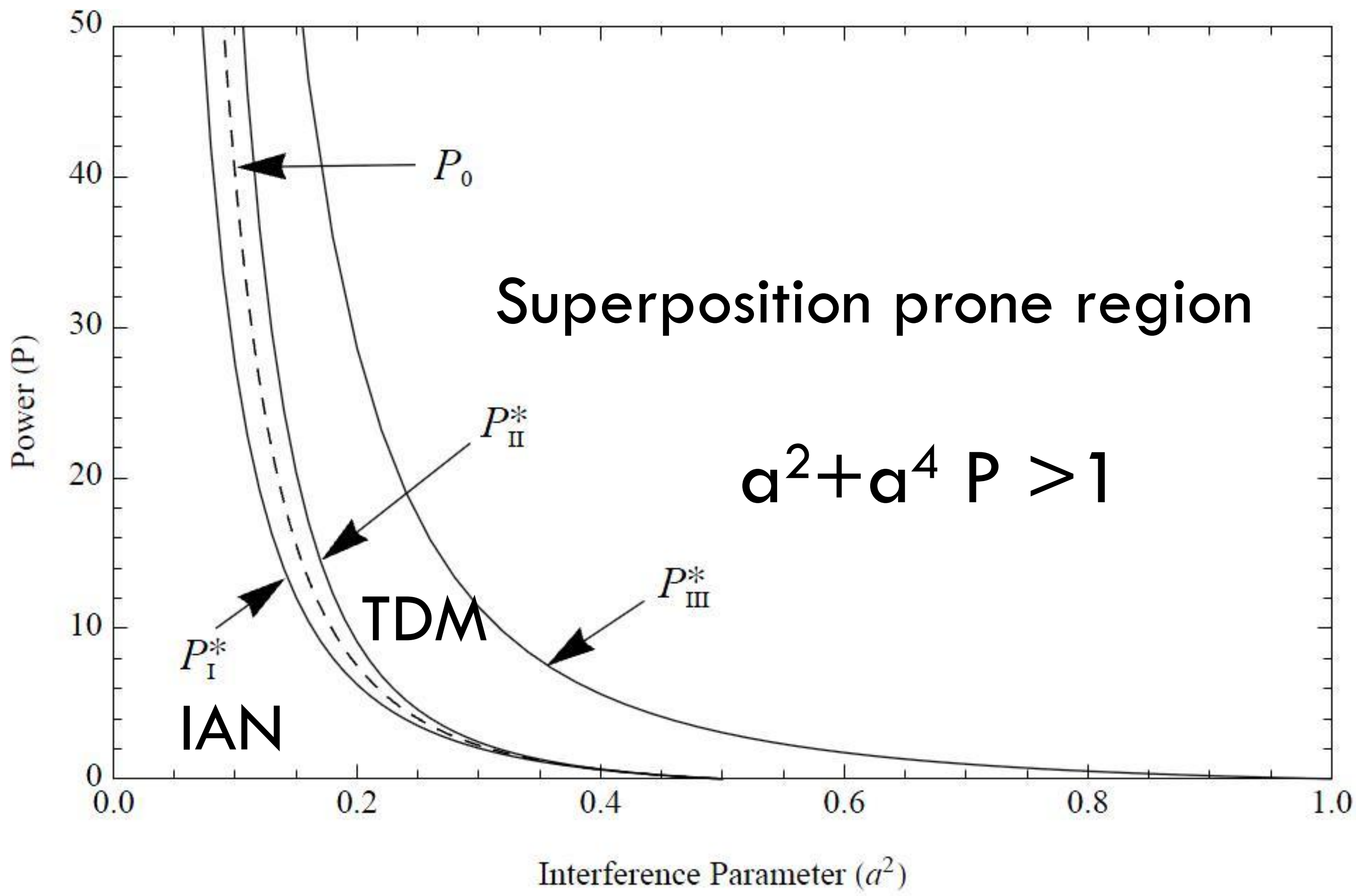
Rate Sum



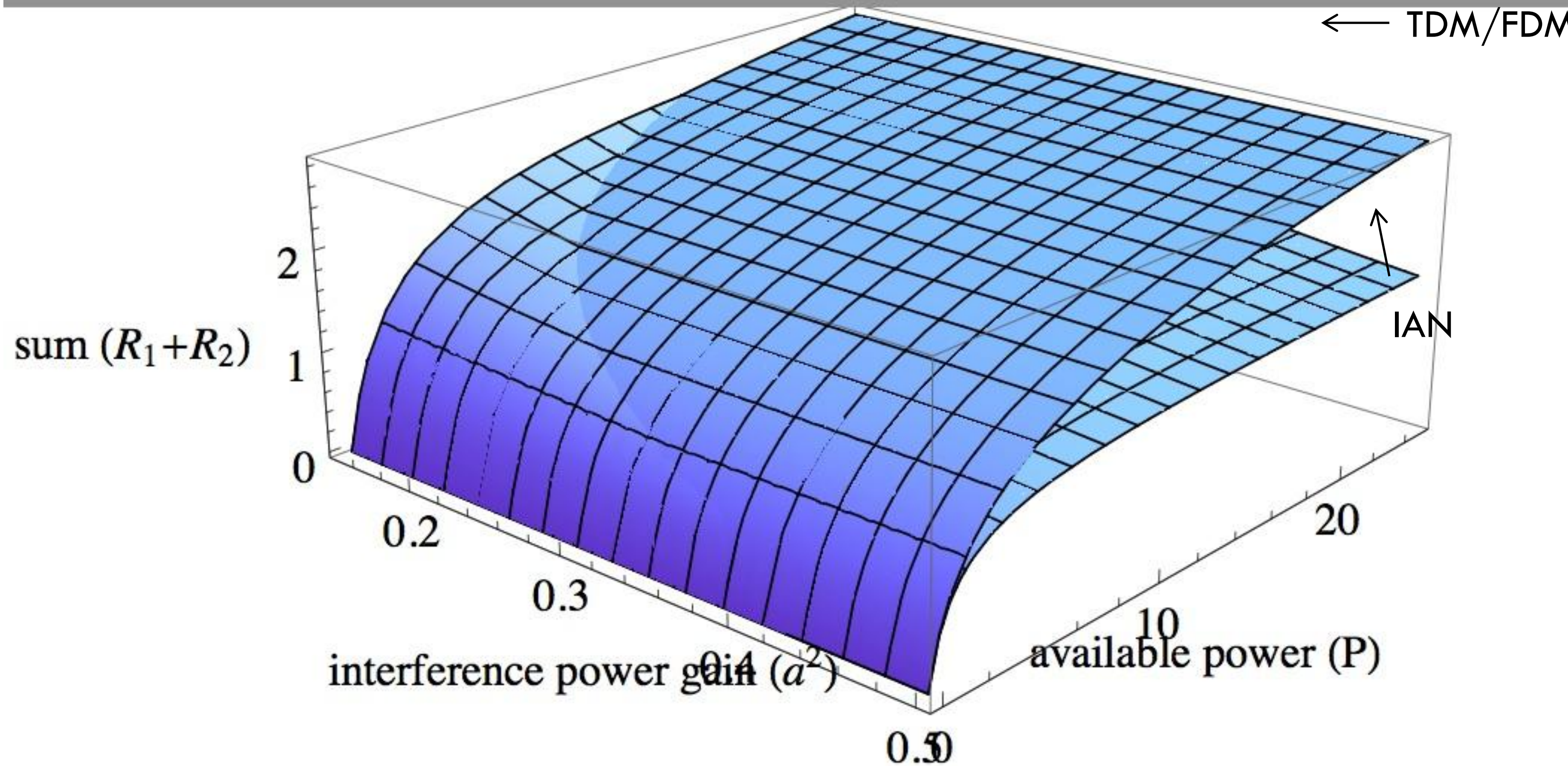
Power P

No intersection beyond $a^2=0.5$

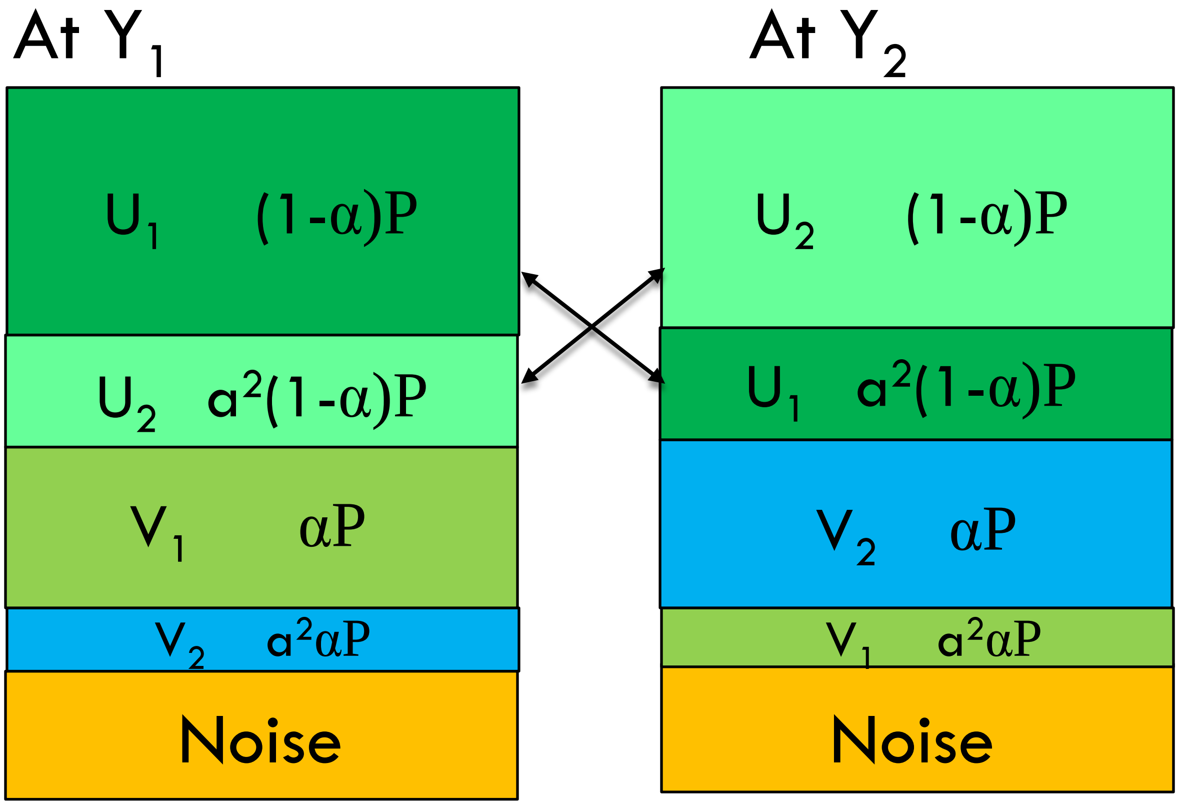
Interference as Noise and TDM/FDM



Rate Sum for IAN and TDM/FDM

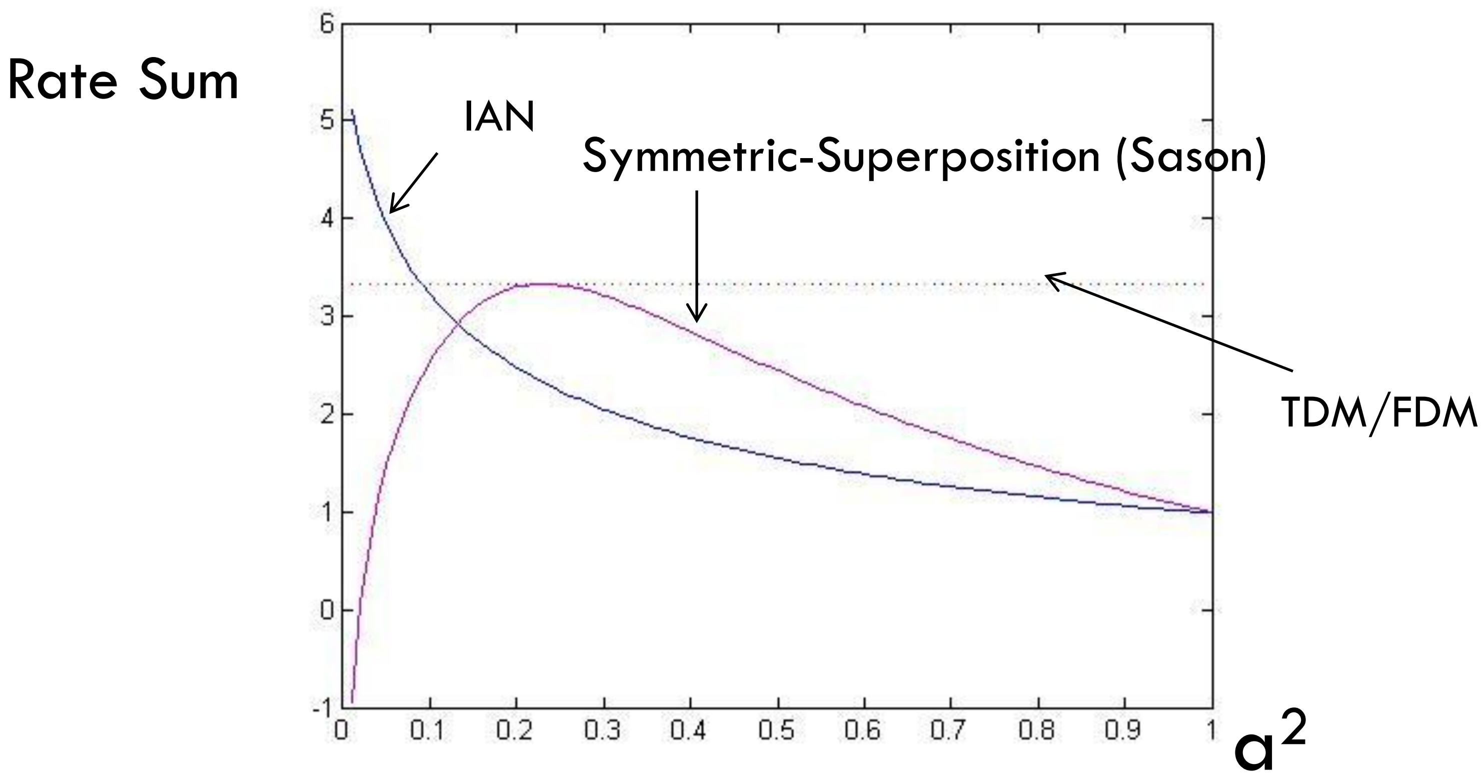


Superposition: partially decoding

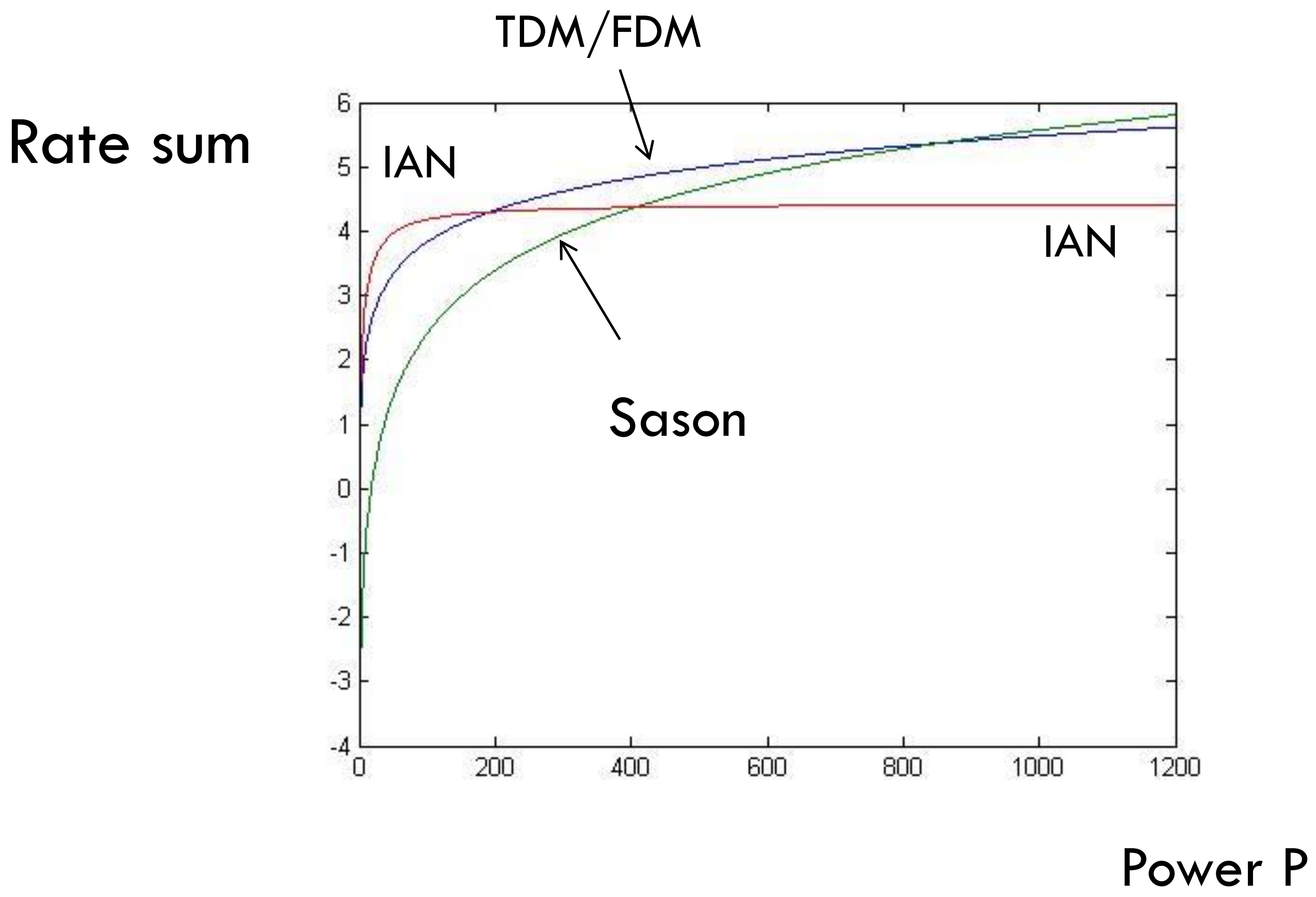


$$R_1+R_2 \leq \log\left(\frac{1+P+a^2P}{\frac{1-a^2}{a^2}+a^2(1+a^2P)}\right)$$, Sason (2004)

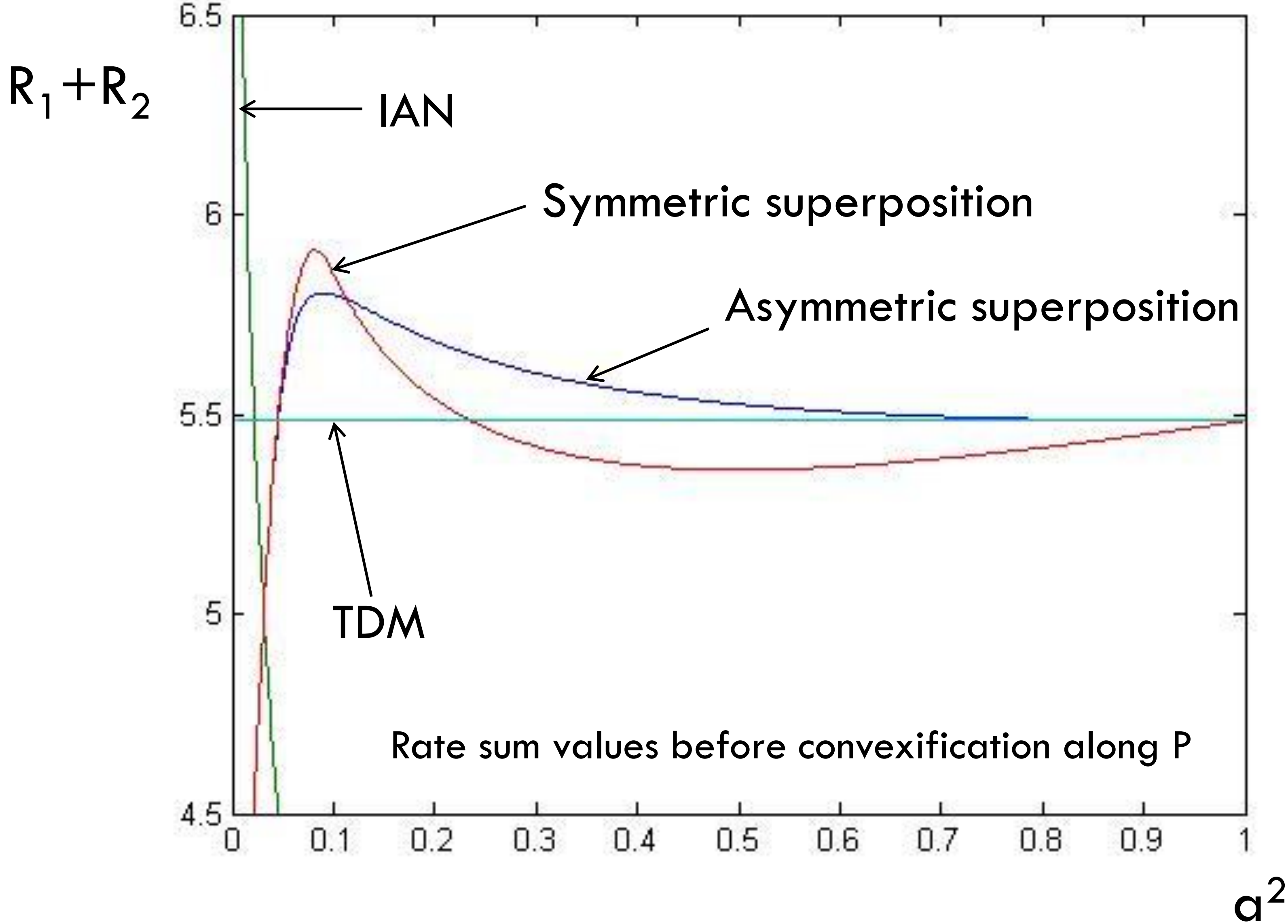
Point where Symmetric Superposition starts beating TDM/FDM
P=50



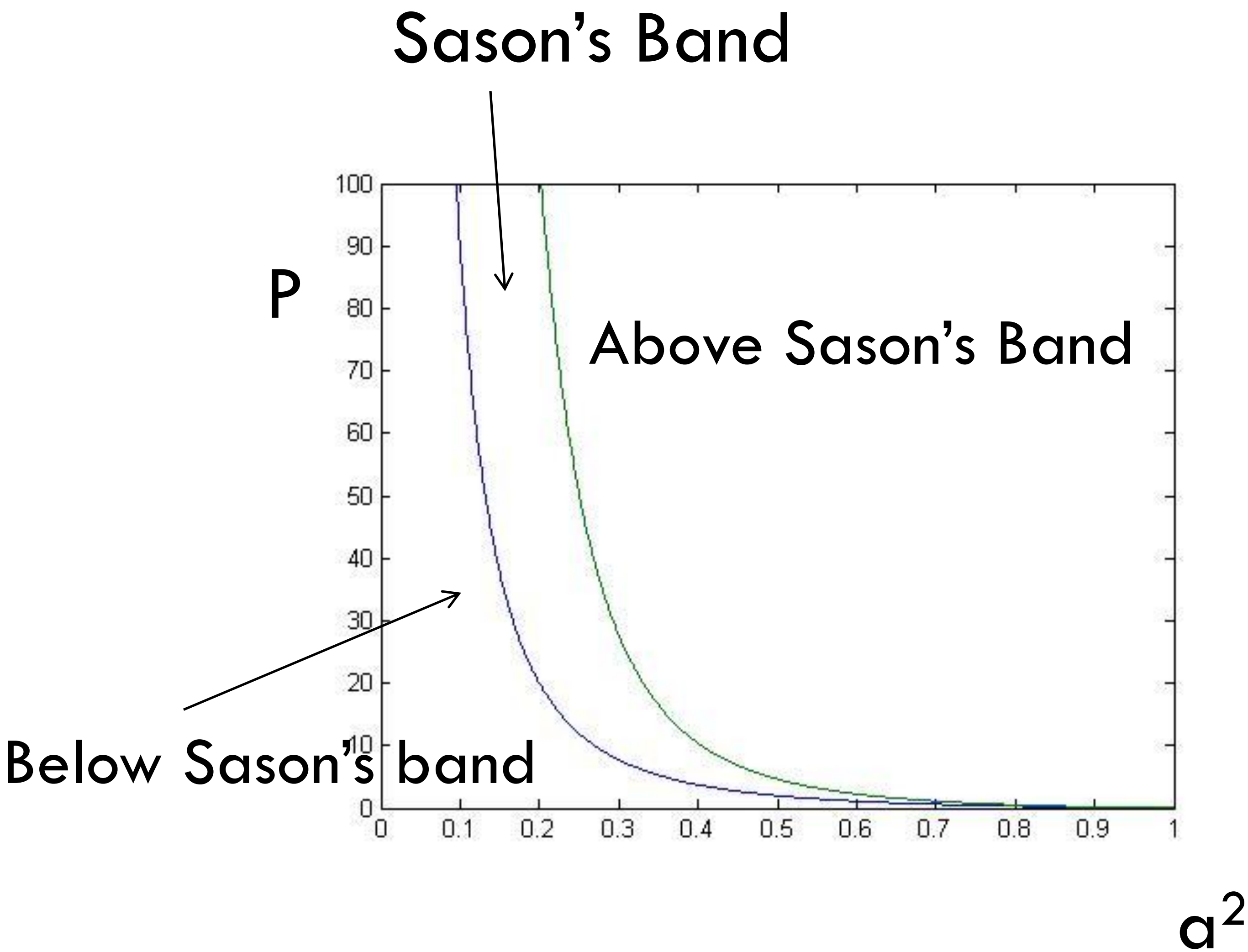
Rate Sum, a^2=0.05: Need convexification



Rate sum for P=1000, $0 \leq a^2 \leq 1$



Symmetric superposition:



Symmetric Superposition (continued):

- **Optimal choice for $\alpha = \alpha_1 = \alpha_2$:**

- Case 1:
- If $\frac{(1-a^2)}{a^4} \leq P \leq \frac{(1-a^6)}{a^6(1-a^2)}$ $(Sason's\ Band)$

  then set $\propto P = a^2(1 + a^2 P) - 1;$

- Case 2:
- If $P \geq \frac{(1-a^6)}{a^6(1-a^2)}$ $(Above\ Sason's\ Band)$

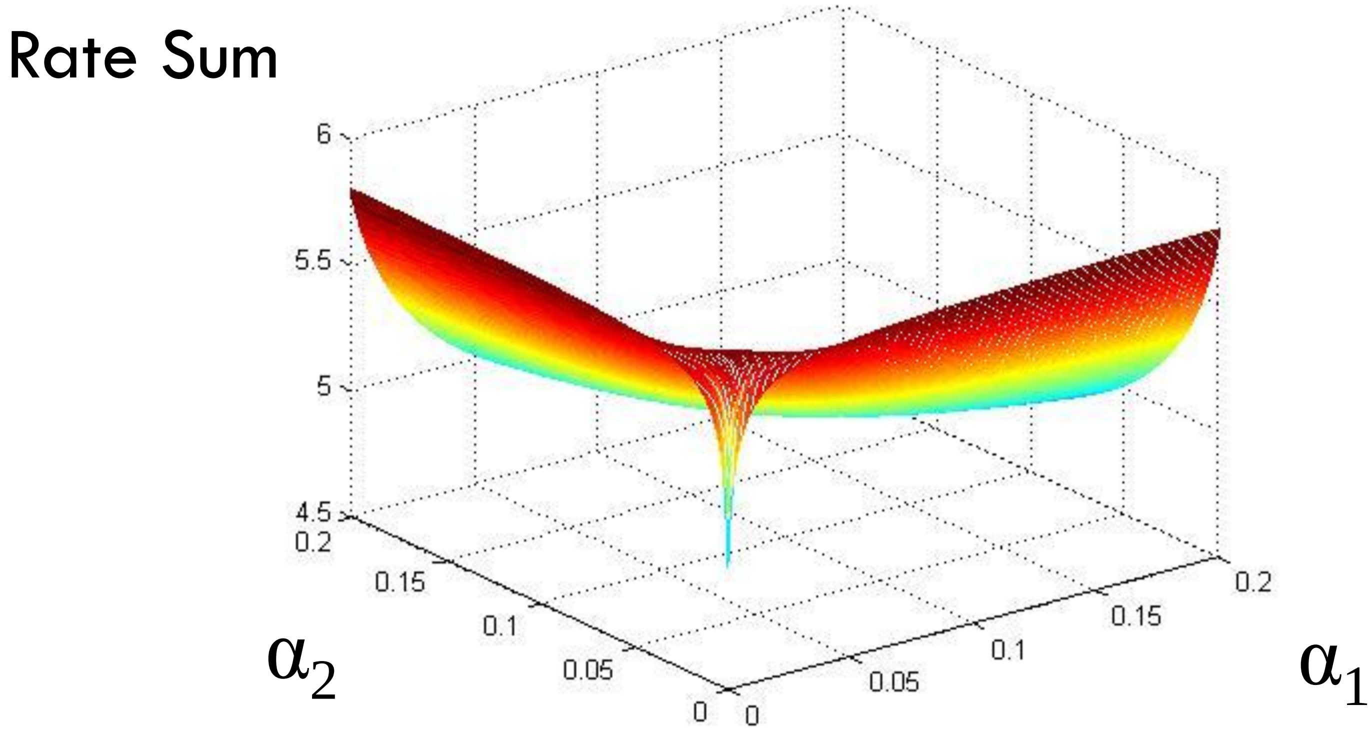
  then set $\propto P = \frac{(1-a^2)}{a^2(1+a^2)}.$

Note: Invariant with P

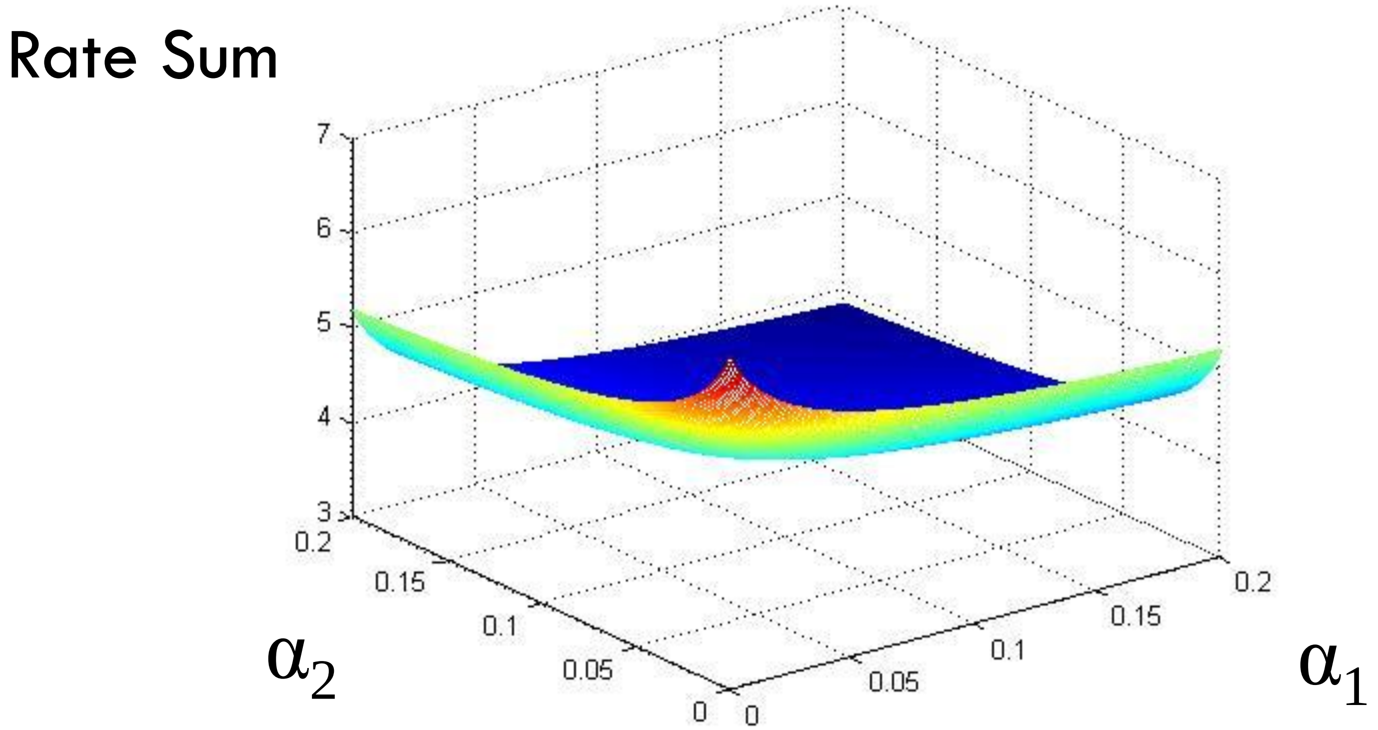
Symmetric Superposition (continued):

- In Sason's Band:
- $$R_1+R_2 \leq \log\left(\frac{a^2\left(1+P+a^2P\right)}{1-a^2+a^4(1+a^2P)}\right)$$
- Above Sason's Band:
- $$R_1+R_2 \leq \frac{1}{2}\log\left(\frac{\left(1+a^2\right)^2\left(1+P+a^2P\right)}{4a^2}\right)$$

The hummingbird function:

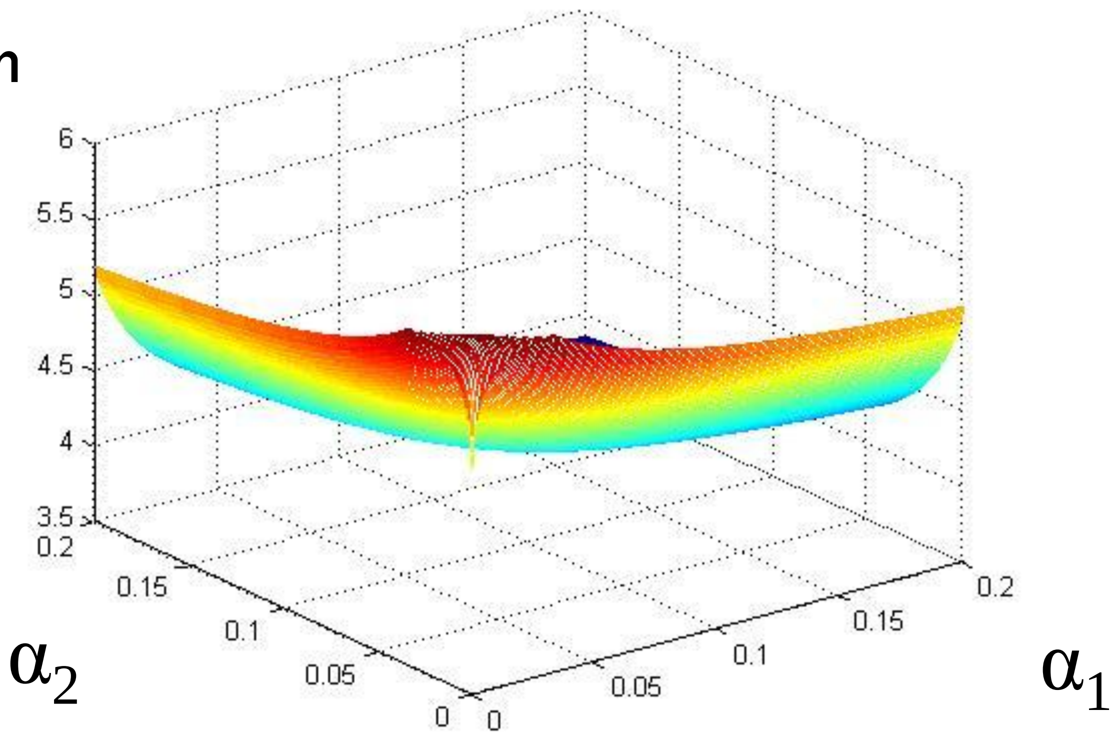


The shroud function

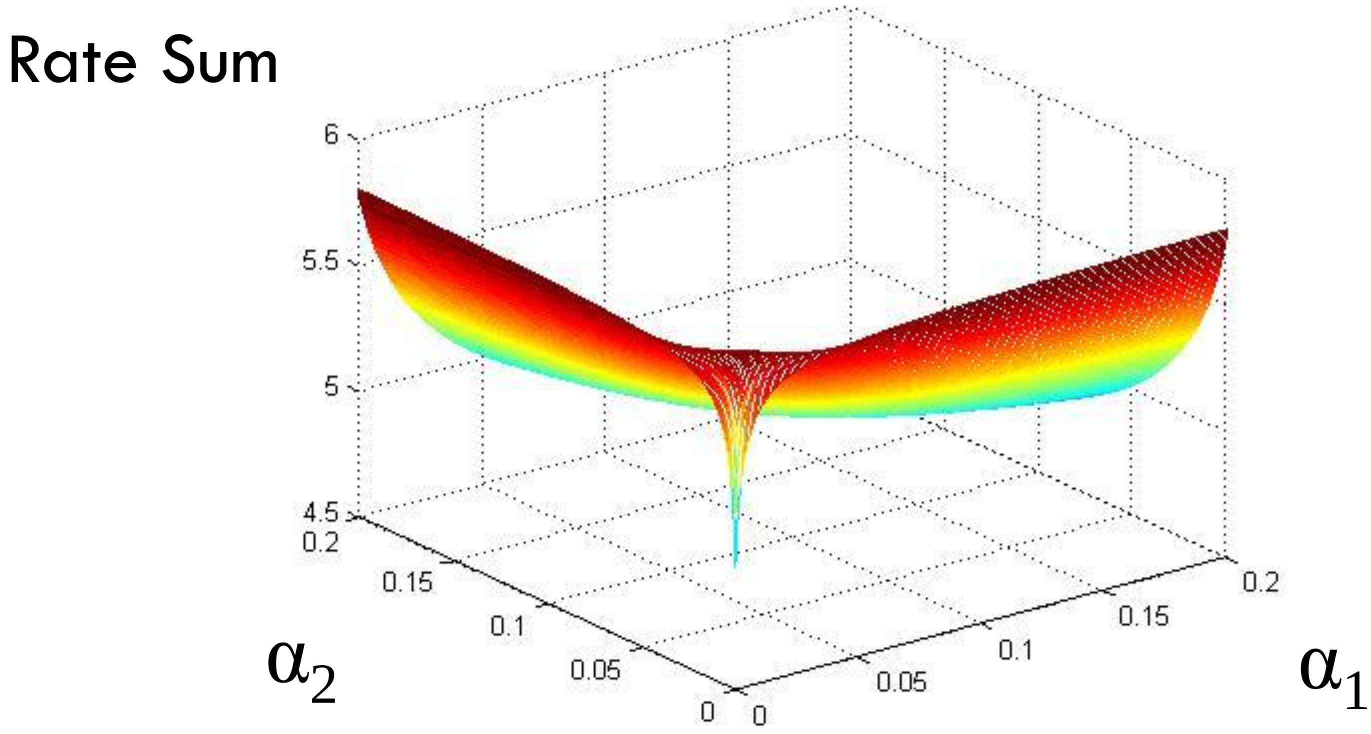


Min (hummingbird, shroud)

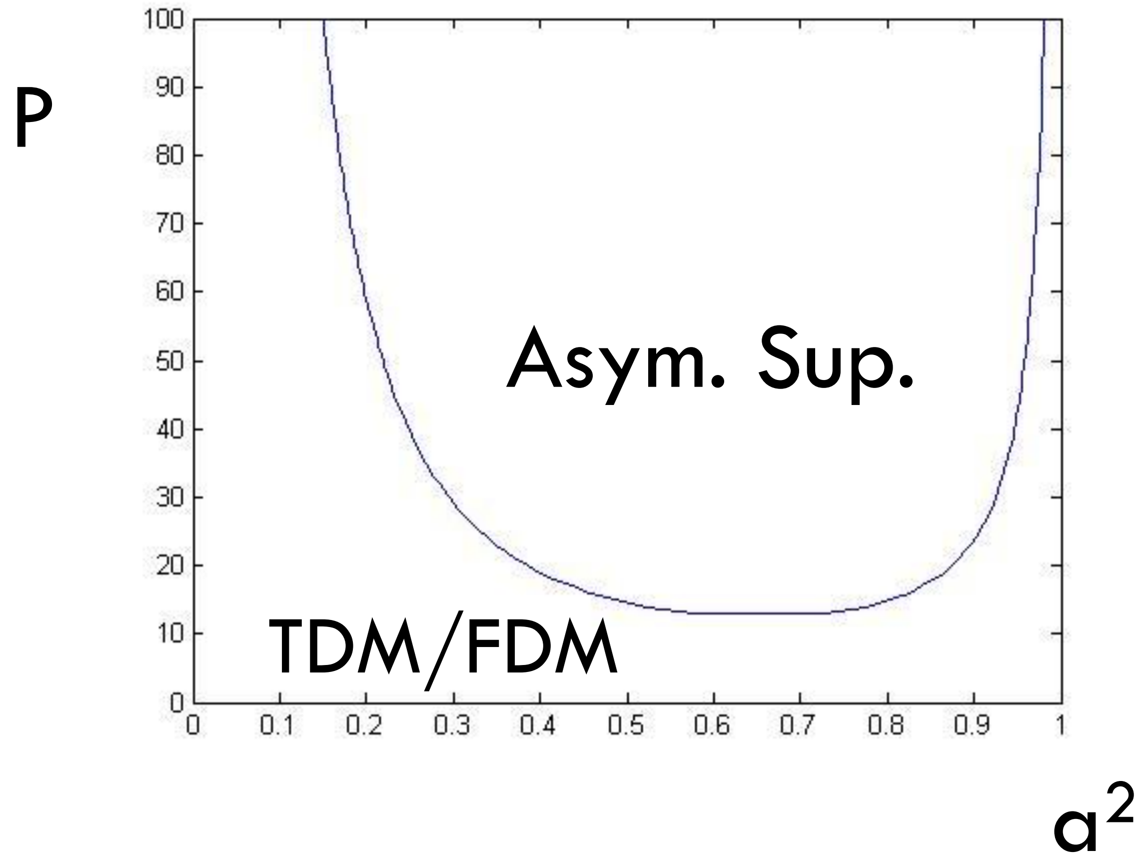
Rate Sum



Flapping wings

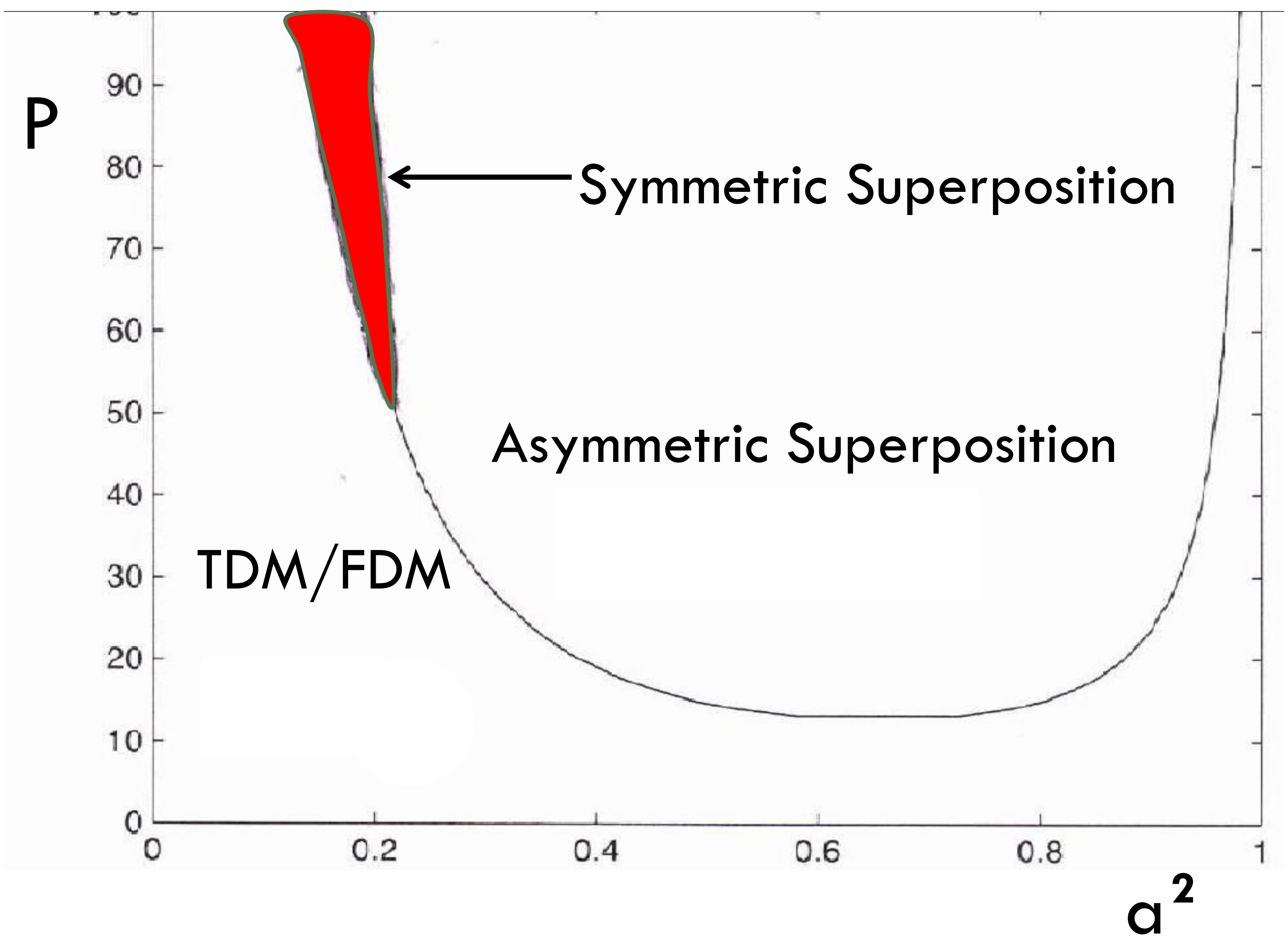


Asymmetric-Superposition vs TDM/FDM

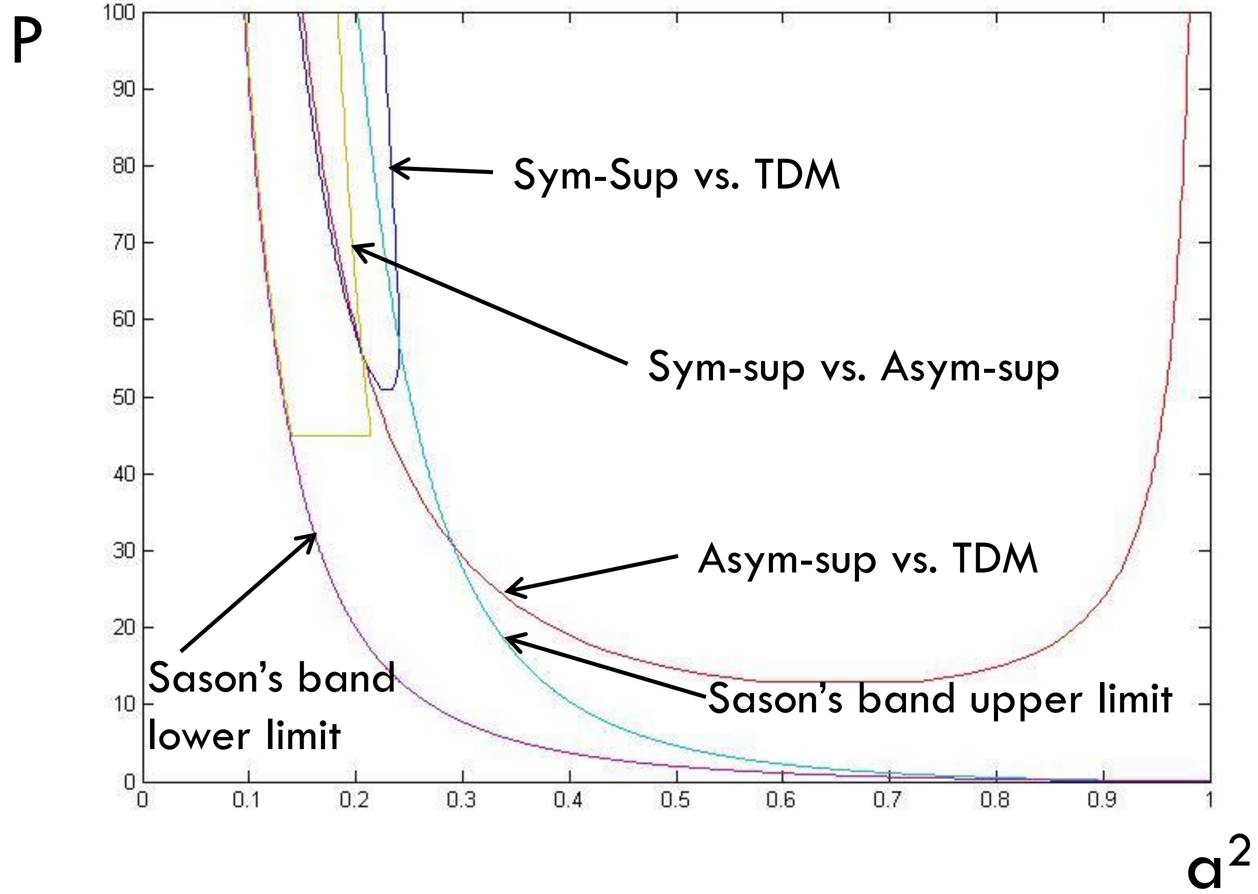


Phase Transitions in Weak Interference

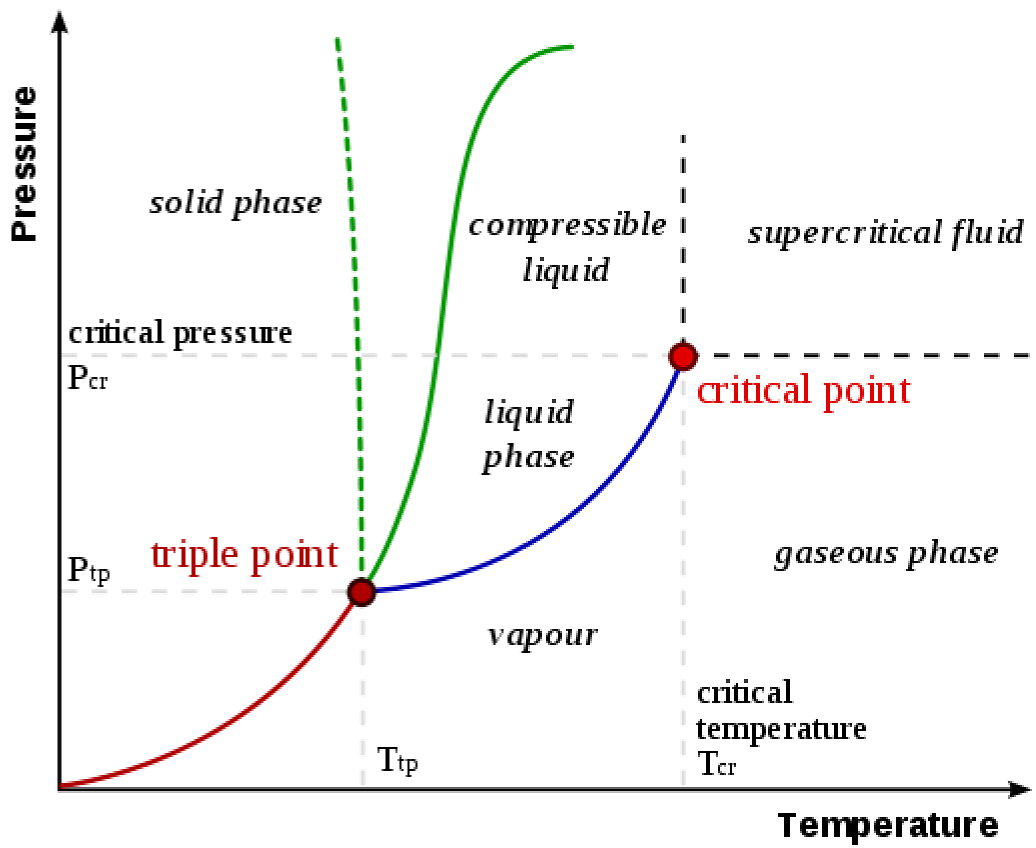
Note: Transitional regions due to convexification along P not included.



Pairwise Phase Transitions



A pleasant resemblance

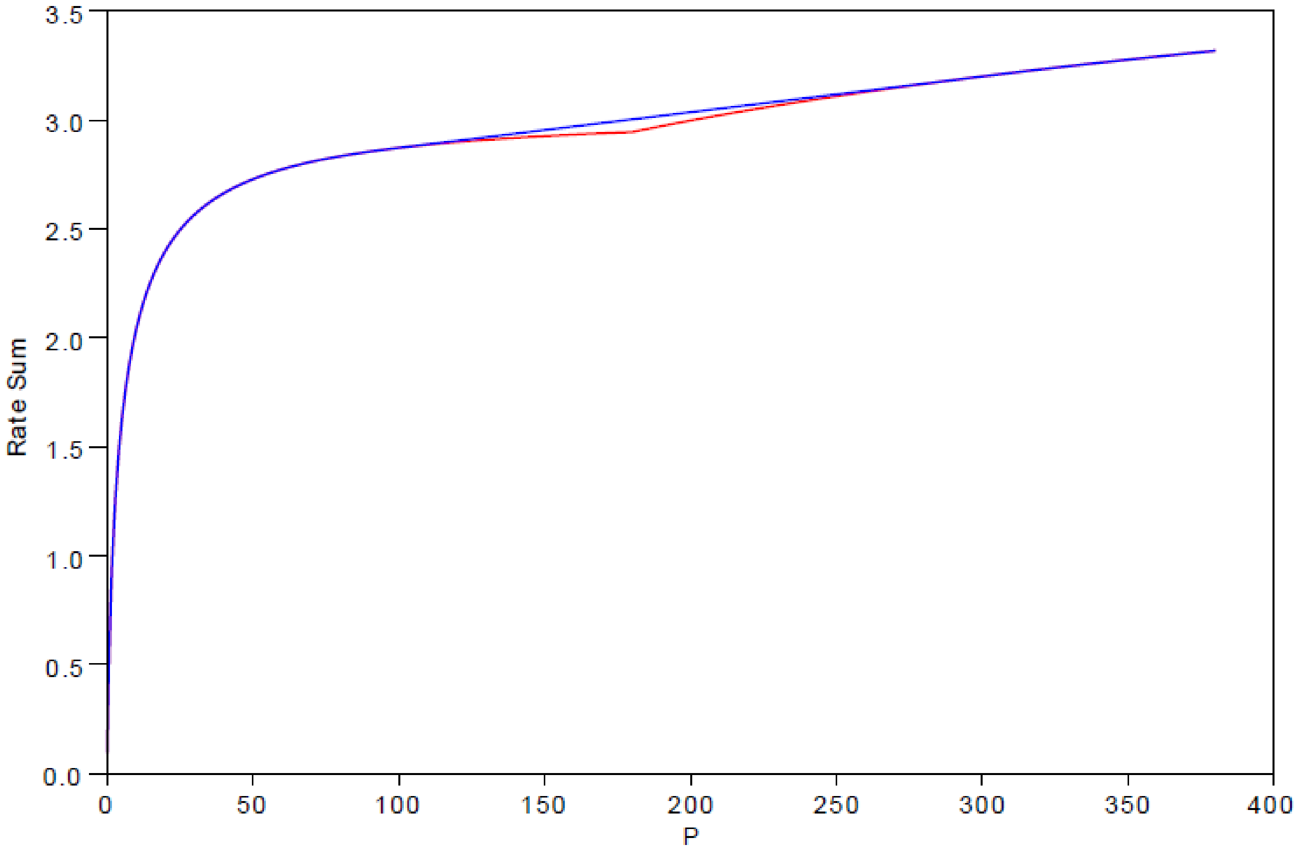


Asymptotically as $P \rightarrow \infty$

$0 < a^2 < 0.087$ -- symmetric superposition is best

$0.087 < a^2 < 1$ – asymmetric superposition is best

As before: Need convexification along P



Final remarks

- Powerful tool: Concave envelopes to transition from one mode to another: time sharing between modes

- Shown a full taxonomy of phase transitions in (a^2, P) parameter space with $0< a^2 <1$, $P>0$:
- 4 pure modes (IAN, TDM, Symmetric Superposition, and Asymmetric Superposition) and
- 4 transitional regions (IAN vs. TDM, TDM vs. Sym-Sup, TDM vs. Asym-Sup, and Sym-Sup vs. Asym-Sup)

Final remarks

- Working to show this is Gaussian Han-Kobayashi Region

- Future directions: Full achievable region
- Show Gaussian signaling is best
- Converse (capacity region)

- Look at general parameter space (P_1, P_2, a, b)

- Many thanks!