Communication Rates for Phase Noise Channels

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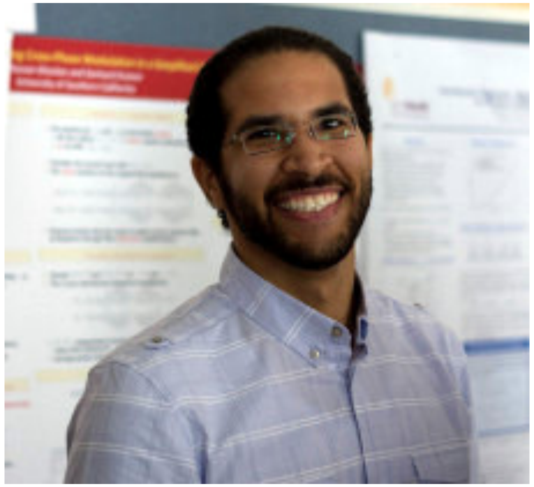
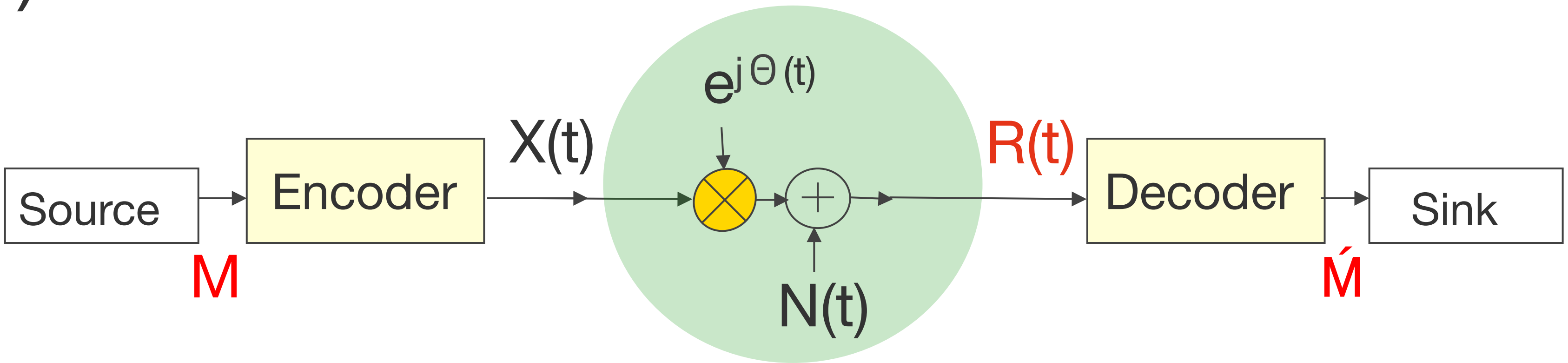
Outline

1) Models
2) Information rates: high-SNR bounds
3) Information rates: numerical methods
4) Examples



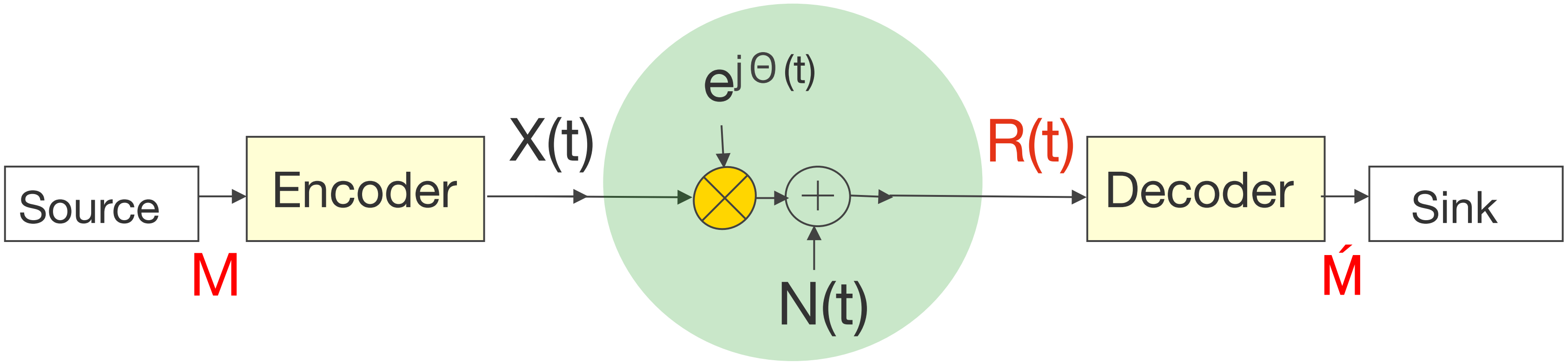
The guy who did most of the work

1) Models



Phase noise affects all oscillators, e.g., for radio and light (laser) communication

- Phase noise statistics depend on the application/receiver
  - phase-locked loops (PLLs) residual noise: von Mises/Tikhonov distribution
  - satellite (DVB-S2): white Gaussian process filtered by IIR filters
  - fiber-optic lasers: Wiener process



- The continuous-time Wiener phase-noise model is

$$R(t) = X(t) \cdot e^{j\Theta(t)} + N(t)$$

where $\Theta(t)$ is a Wiener process

$$\Theta(t) = \Theta(0) + \int_0^t W(\tau)\, d\tau$$

and N(t), W(t) are white Gaussian, $\Theta(0)$ uniform on $[0,2\pi)$

- NB: both white processes defined via integral (filter) equation

Wiener Phase Noise Statistics

- Autocorrelation function for white noise*

$$E\left[N(t_1)N^*(t_2)\right] = \sigma_N^2\,\delta(t_1 - t_2)$$

$$E\left[W(t_1)W^*(t_2)\right] = 2\pi\beta\,\delta(t_1 - t_2)$$

- Autocorrelation & PSD function of U(t)=exp(j Θ (t))

$$R_U(t_1,t_2) = E\left[U(t_1)U^*(t_2)\right] = \exp\left(-\pi\beta\left|t_2 - t_1\right|\right)$$

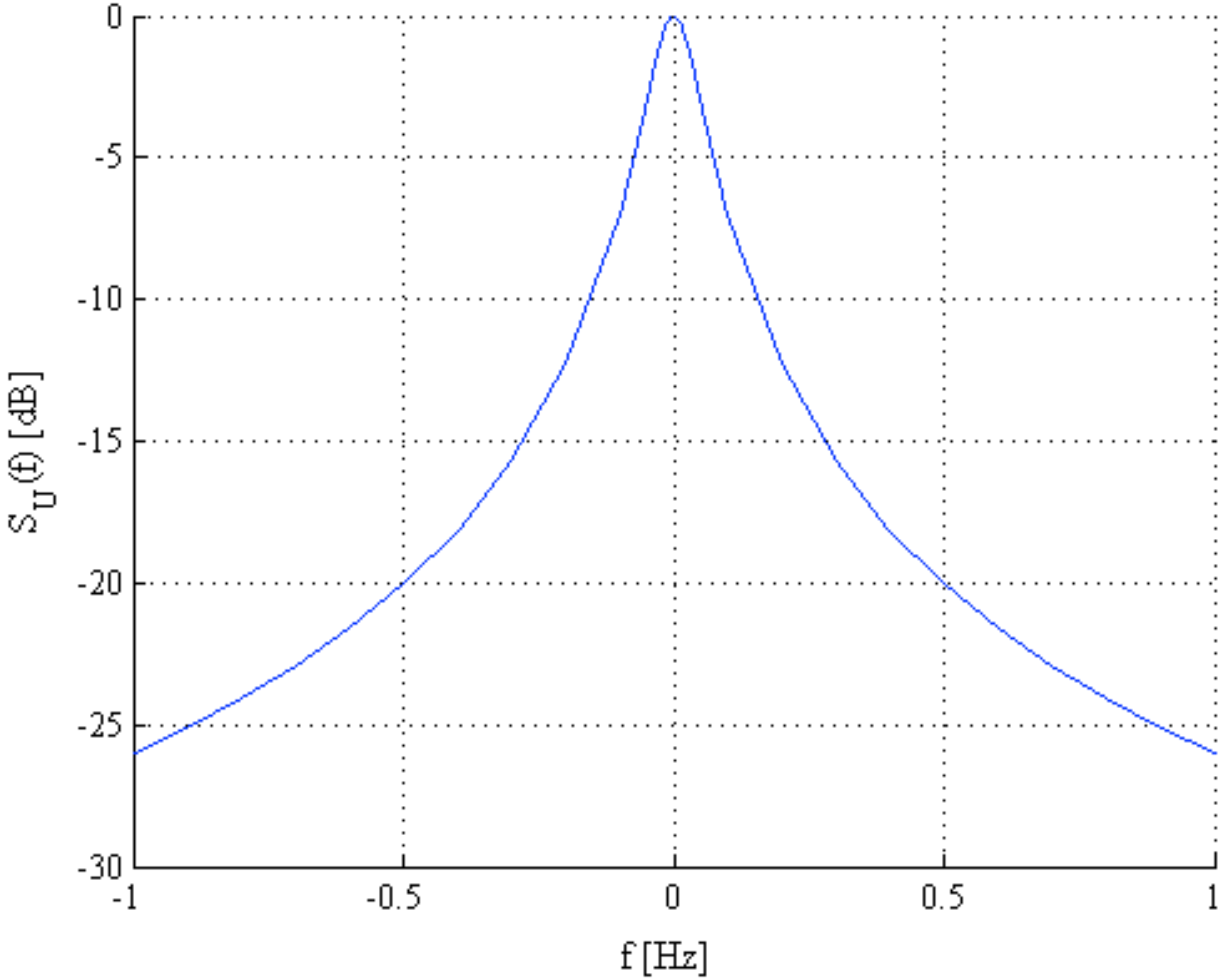
$$S_U(f) = \frac{1}{\pi}\frac{\beta/2}{(\beta/2)^2 + f^2}$$

- PSD is Lorentzian: β is "full-width at half-maximum" or twice the "half-width at half-maximum"

* Generalized process defined by an integral equation: a filter

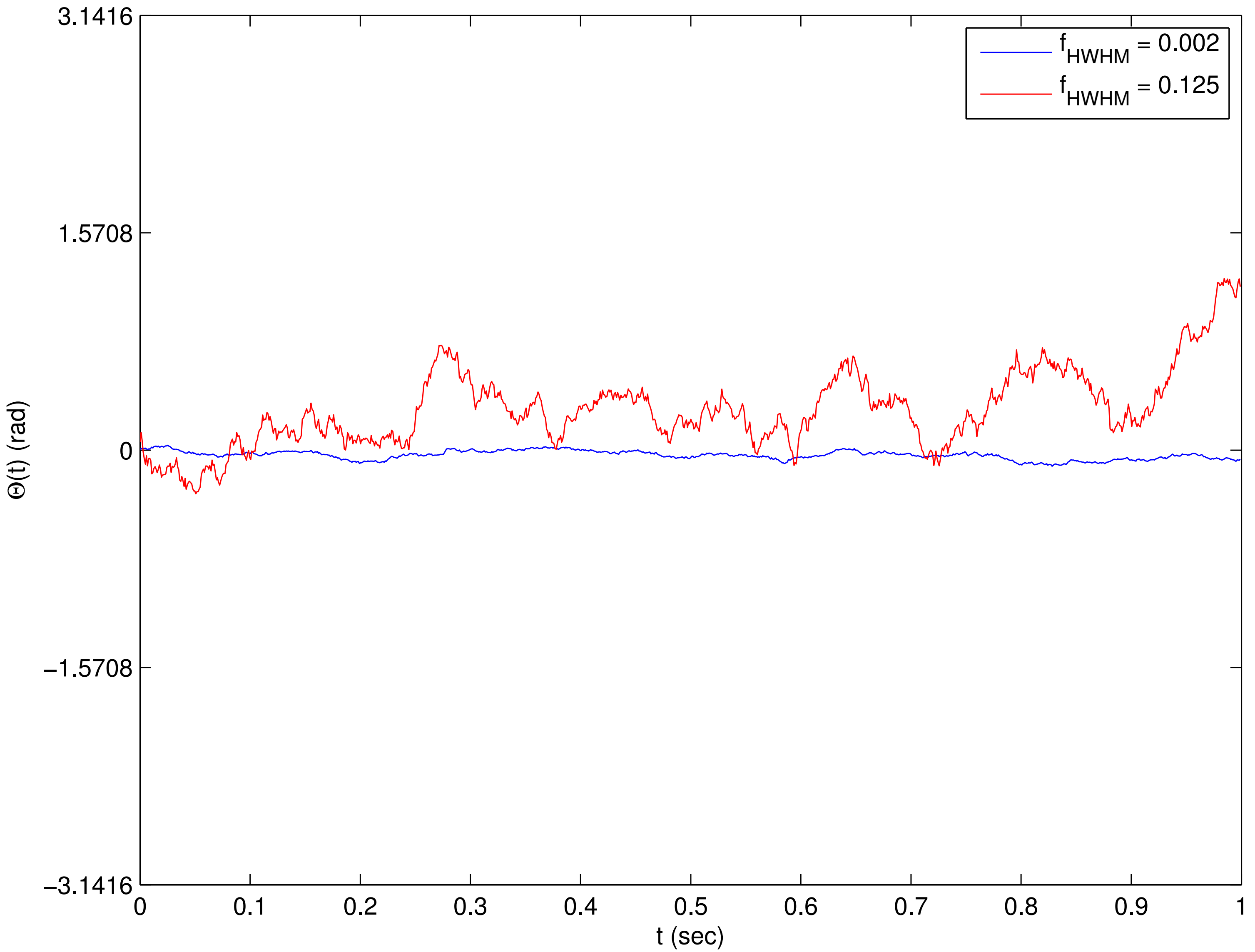
Lorentzian PSD

- βT_{symb}=0.1 where T_{symb} is the symbol (or sampling) time
- PSD and f are normalized
- PSD is for multiplicative noise: convolution of spectra
- Infinite bandwidth expansion*



* Causes aliasing and "spectral loss"

Wiener Process Sample Paths



Some Literature on (Wiener) Phase Noise in Communications

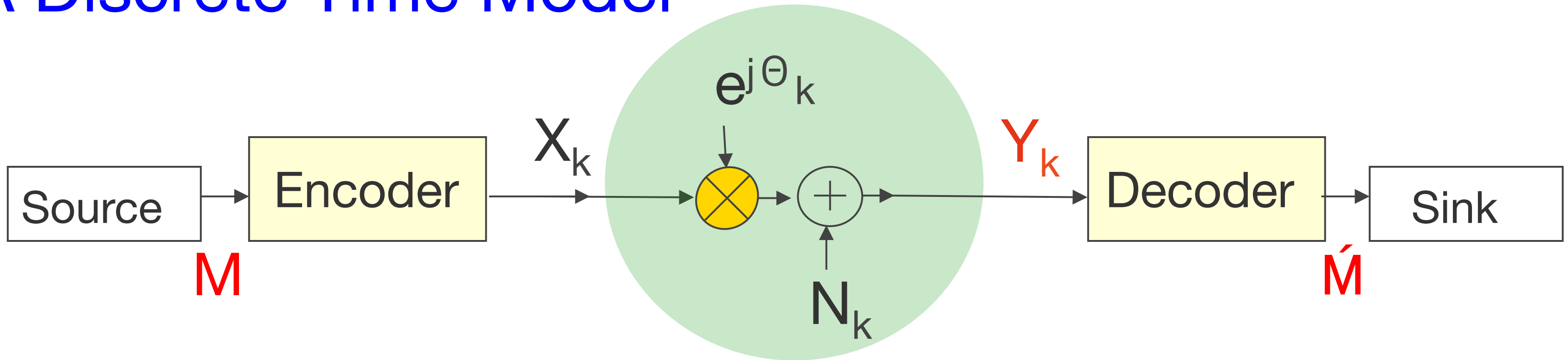
- 1980s - early 1990s; attention was on optical (coherent detection for single-mode fiber + laser phase noise)
  - 1986: Jeromin-Chan, Kazovsky, Salz
  - 1988: Foschini-Vannucci, **F-V-Greenstein***, Okoshi-Kikuchi, Wu-Wu
  - 1989: Dallal-Shamai, F., Garret-Jacobsen, Greenstein-V.-F., Linke
  - 1990: Castagnozzi, Cimini-Foschini, Dallal-Shamai, Barry-Lee, Garret, Kazovsky-Toguz, Tsao
  - 1991-94: Azizoglu-Humblet, Dallal-Shamai, Nassar-Soleymani
  - 2000: Peleg-Shamai-Galan
- Analysis is difficult due to filtering and memory

* Studies oversampling

- Why focus on Wiener phase noise?
  - a single parameter (the noise variance) process with two important characteristics: continuous-time and memory
  - gives insight on behavior of other filtered processes
- For simplicity, we consider receiver phase noise only; there is usually also transmitter phase noise
- Wireless: phase noise power is often considered small
  Common approach: ignore, or treat discrete-time phase noise*
- Question: when are these approaches accurate?

* Lots of recent papers

A Discrete Time Model



- A commonly* used discrete-time model

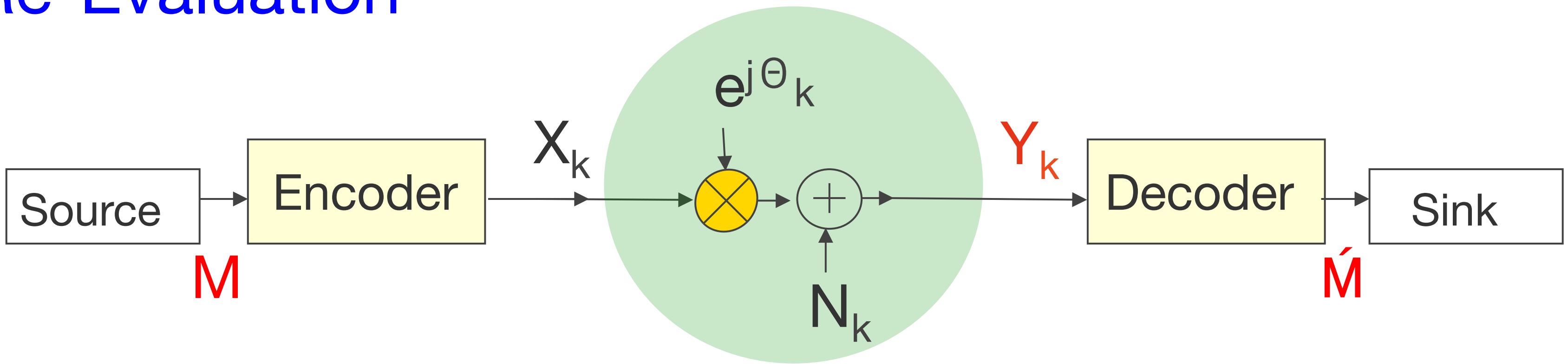
$$Y_k = X_k \cdot e^{j\Theta_k} + N_k$$

for k=1,2,…,n where

  - $\{\Theta_k\}$ is discrete time Wiener with $\Theta_k = \Theta_{k-1} + W_k$
  - $\{W_k\}$ and $\{N_k\}$ are white Gaussian processes

* Most papers since 1994; almost all wireless papers

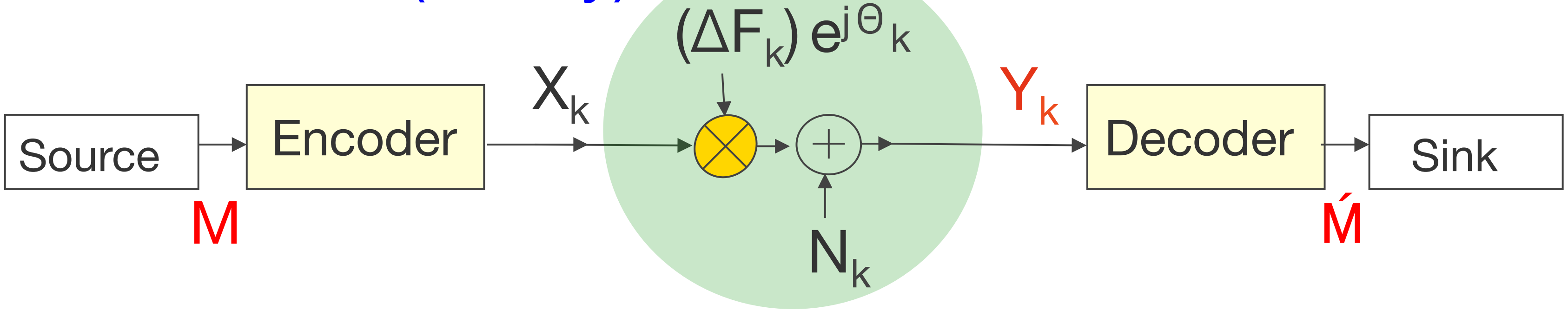
Re-Evaluation



- Is this a good model?
- Quick Answer: yes, if $\beta\, T_{symb}$ is small*
- Refined Answer: yes, if $\beta\, T_{symb}$ small **and SNR not too large**
- Reason: to discretize one must filter which converts phase noise into both phase and amplitude noise

* Fine for many situations

Discretization (2nd try)



- Receiver (integrate & dump): $Y(t) = \int_{t-\Delta}^{t} \left[X(\tau) e^{j\Theta(\tau)} + N(\tau) \right] d\tau$
- Samples for <u>square</u> pulse shape* (with $\Delta = T_{symb}$)

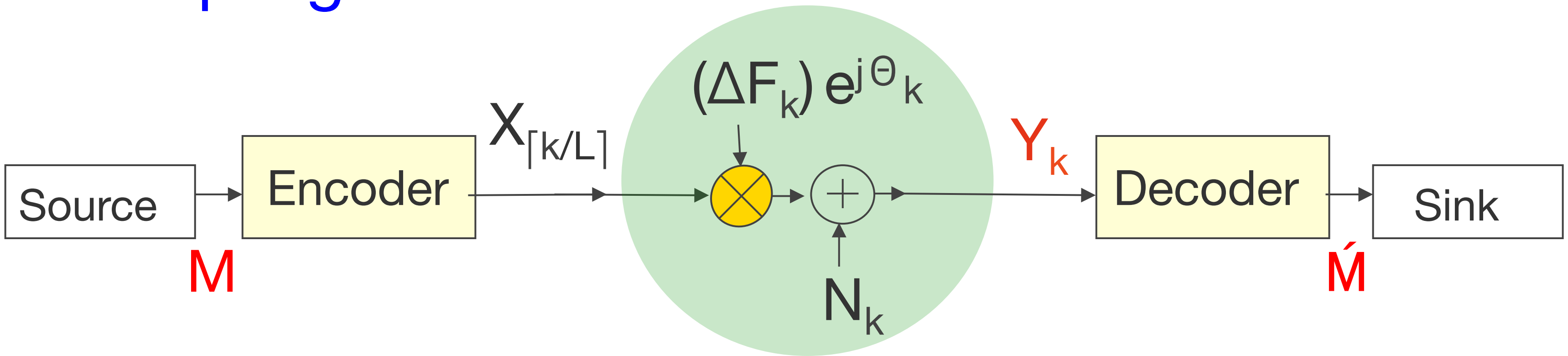
$$Y_k = X_k \cdot (\Delta F_k) \cdot e^{j\Theta_k} + N_k$$

$$F_k = \frac{1}{\Delta} \int_{(k-1)\Delta}^{k\Delta} e^{j(\Theta(t)-\Theta_k)}\, dt$$

where $\{\Theta_k\}$ is discrete-time Wiener with $E[|W_k|^2] = 2\pi\beta\Delta$;
$\{F_k\}$ is an i.i.d. process; F_k and Θ_k are dependent

* Pulse shaping is interesting, e.g., peaky or bandlimited pulses

Oversampling*



- Oversampling (OS) turns out to be important at high SNR**
- OS with $\Delta = T_{symb}/L$:

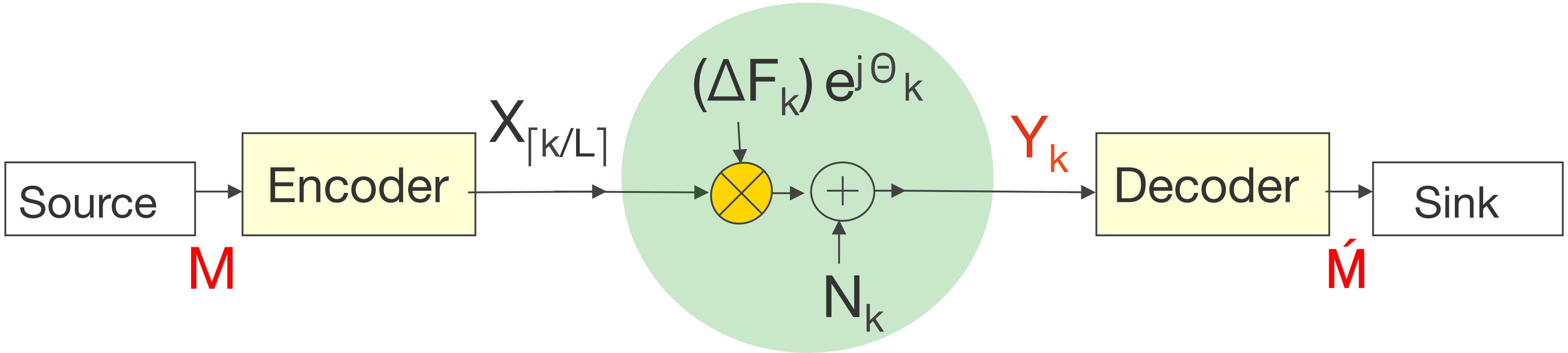
$$Y_k = X_{\lceil k/L \rceil} \cdot \left(\Delta F_k \right) \cdot e^{j\Theta_k} + N_k$$

where N_k has factor L less power than with L=1

- NB: this type of OS requires much “free bandwidth” around the main carrier; not a good fit for spectral efficiency!

* FGV (1988); **Ghozlan & Kramer (2013/2014)

2) Information Rates: High-SNR Bounds



Full Model

- If L=constant then C ~ log log SNR (needs a proof!)
- If L~SNR$^{1/2}$ (or SNR$^{1/3}$) then* C ≥ ½log(SNR) for large SNR

Model with ΔF_k=1

- If L=constant then C~½log(SNR) for large SNR
- If L~SNR$^{1/2}$ then** C ≥ ¾log(SNR) for large SNR**

Ghozlan & Kramer (ISIT 2013*/2014**)

Model with ΔF_k=1

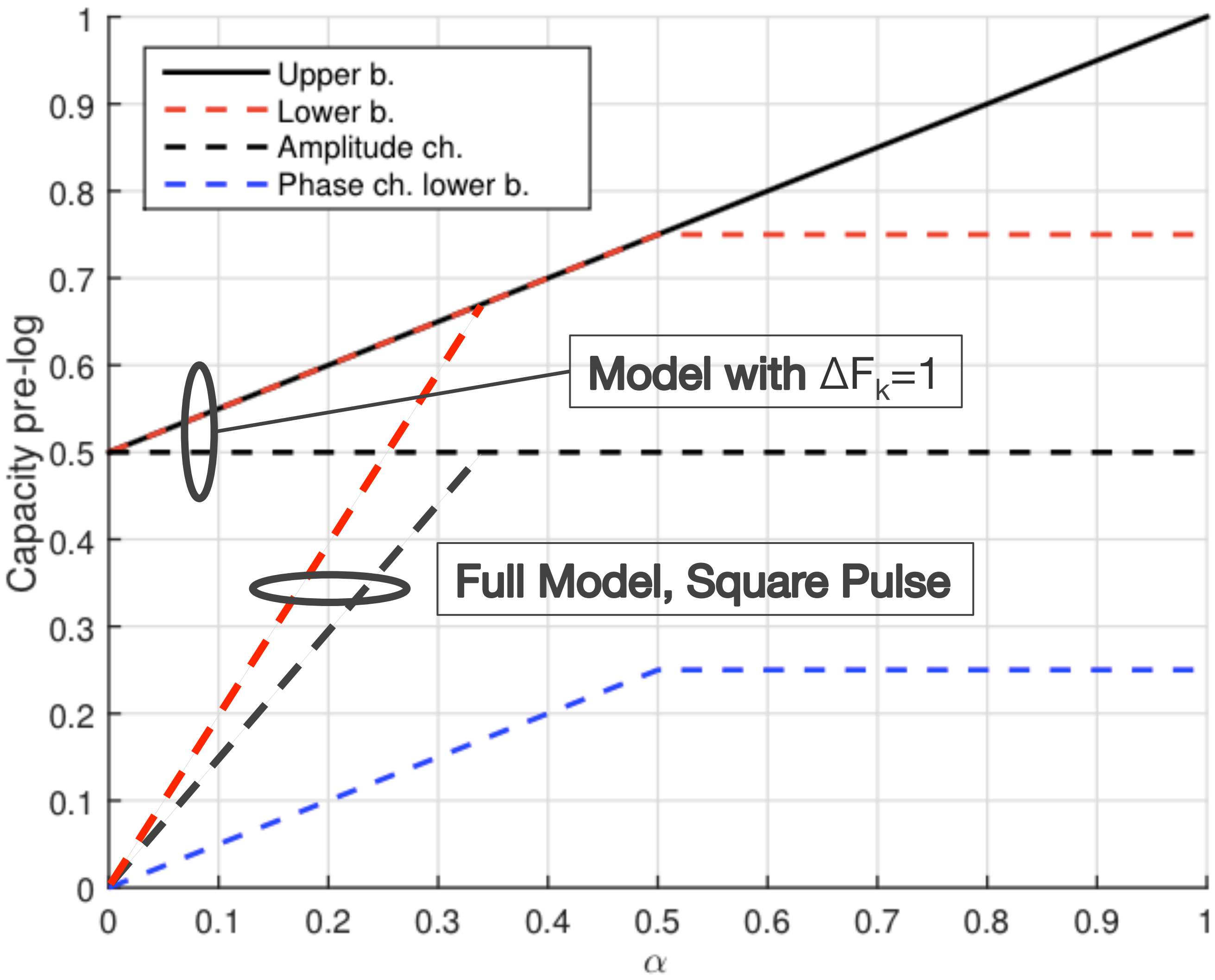
- L~SNR$^{\alpha}$ then*

  C≤(1+α)/2 log(SNR)

  for large SNR
- Examples:
  α=0, ½

Full Model

- lower bound**
- no upper bound yet (other than trivial "1")



* Barletta ITW 2015; ** Barletta ISIT 2015

3) Information Rates: Numerical Methods

Information theory specifies rates for reliable communication

- Let $X = X_1 X_2 \ldots X_n$ and $Y = Y_1 Y_2 \ldots Y_{nL}$
- We wish to compute I(X;Y) for large n where

$$I(X;Y) = \mathrm{E}\left[\log \frac{p(X,Y)}{p(X)p(Y)}\right]$$

Problem 1: p(y|x) is unknown (or hard to compute)

Problem 2: X_k are discrete but Y_k are continuous

- Method 1: use an auxiliary channel lower bound*
- Method 2: discretize Θ_k and track with graphical model

* Arnold et al. (IT Transactions 2006)

1) Auxiliary Channel Lower Bound on Information Rate

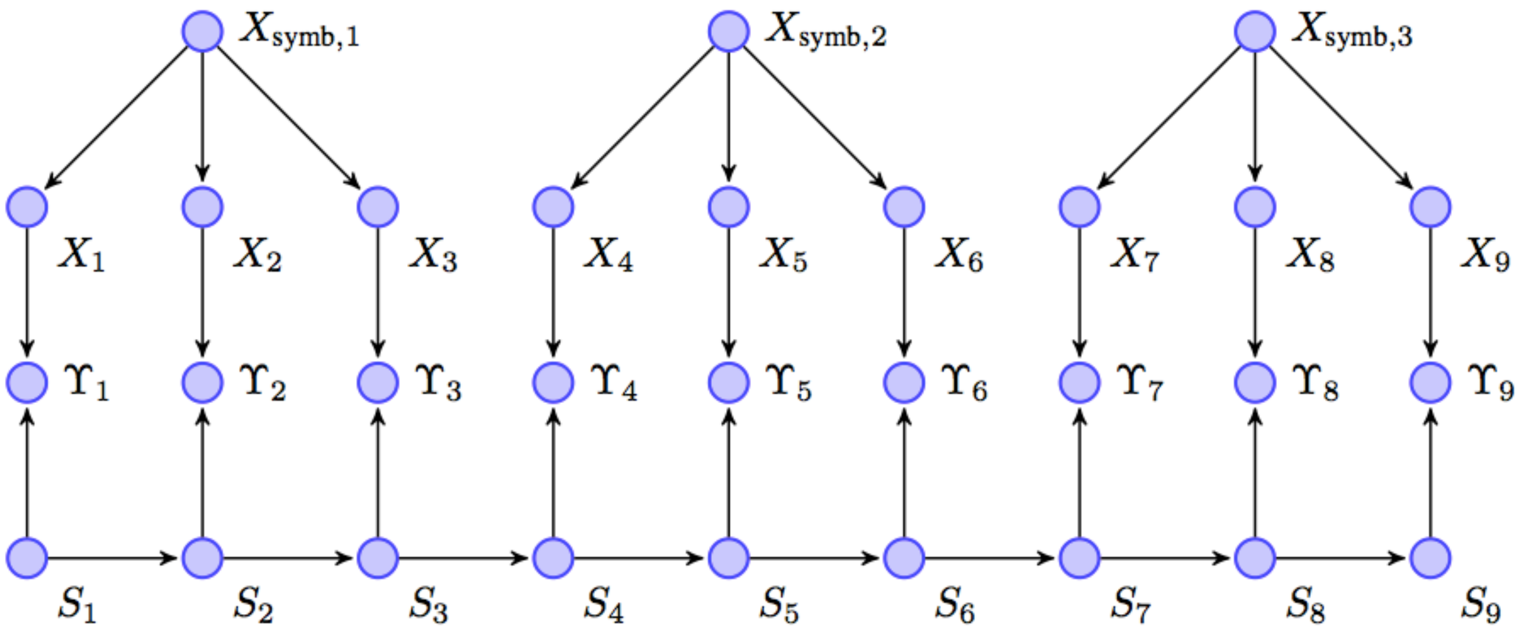
- The mutual information I(X;Y) is lower bounded by

$$I(X;Y) = \mathrm{E}\left[\log \frac{p(X,Y)}{p(X)p(Y)} \cdot \frac{q(Y)}{q(Y|X)} \cdot \frac{q(Y|X)}{q(Y)}\right]$$

$$= D\big(p(X,Y) \,\|\, p(Y) \cdot q(X|Y)\big) + \mathrm{E}\left[\log \frac{q(Y|X)}{q(Y)}\right] \geq \mathrm{E}\left[\log \frac{q(Y|X)}{q(Y)}\right]$$

- Interpretation: choose a q(y|x) that is easy to compute
  - simulate long sequence of XY via actual model p(x,y)
  - compute q(y|x) and q(y) = ∑ p(x) q(y|x)
  - compute the last expectation above as a lower bound

2) Graphical Models



- Example for 3 symbols, OS factor L=3
- For the plots, we consider L=1,2,4,8,16 and discretize the phase to S= 16,32,64 states

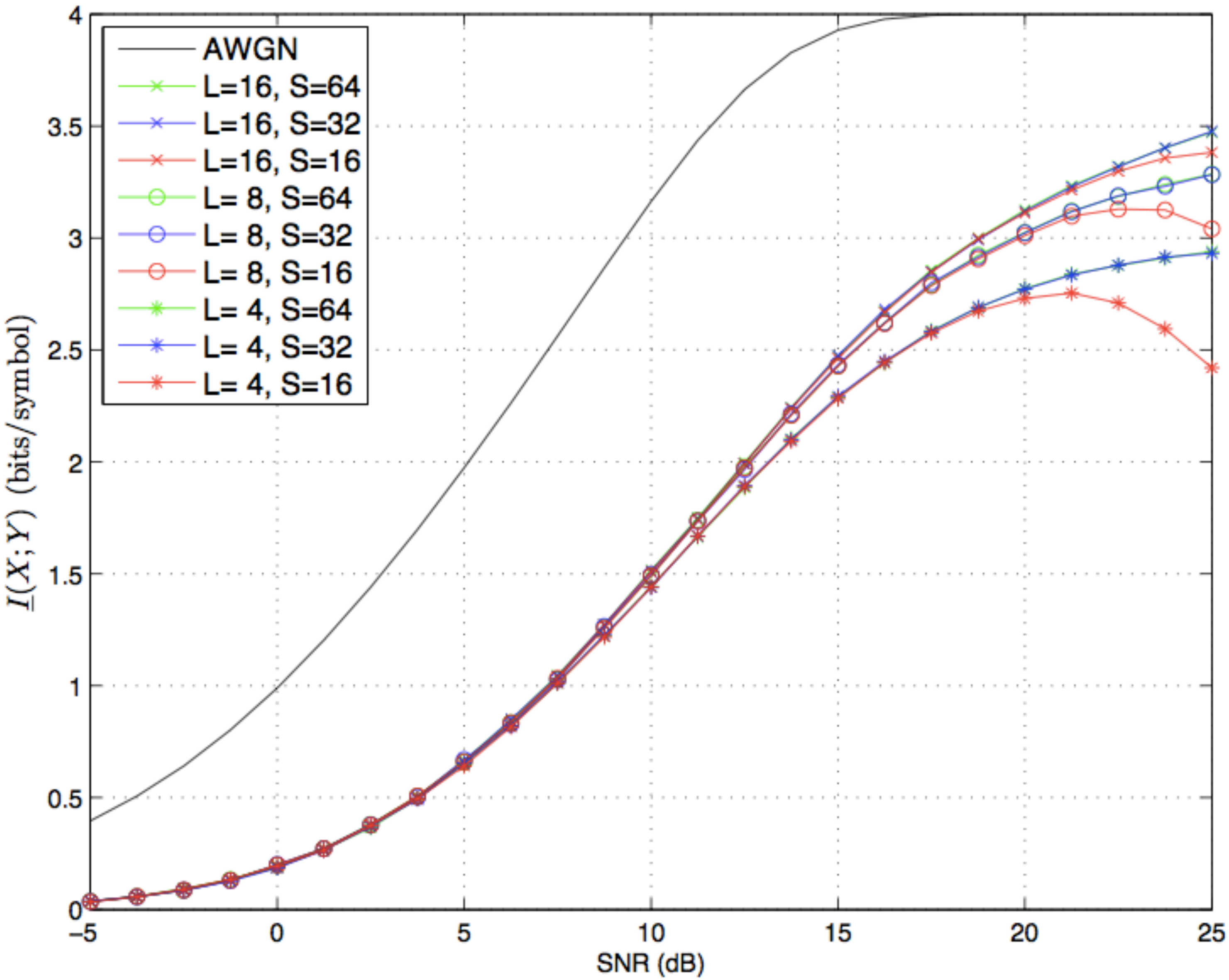
4) Examples

Parameters:

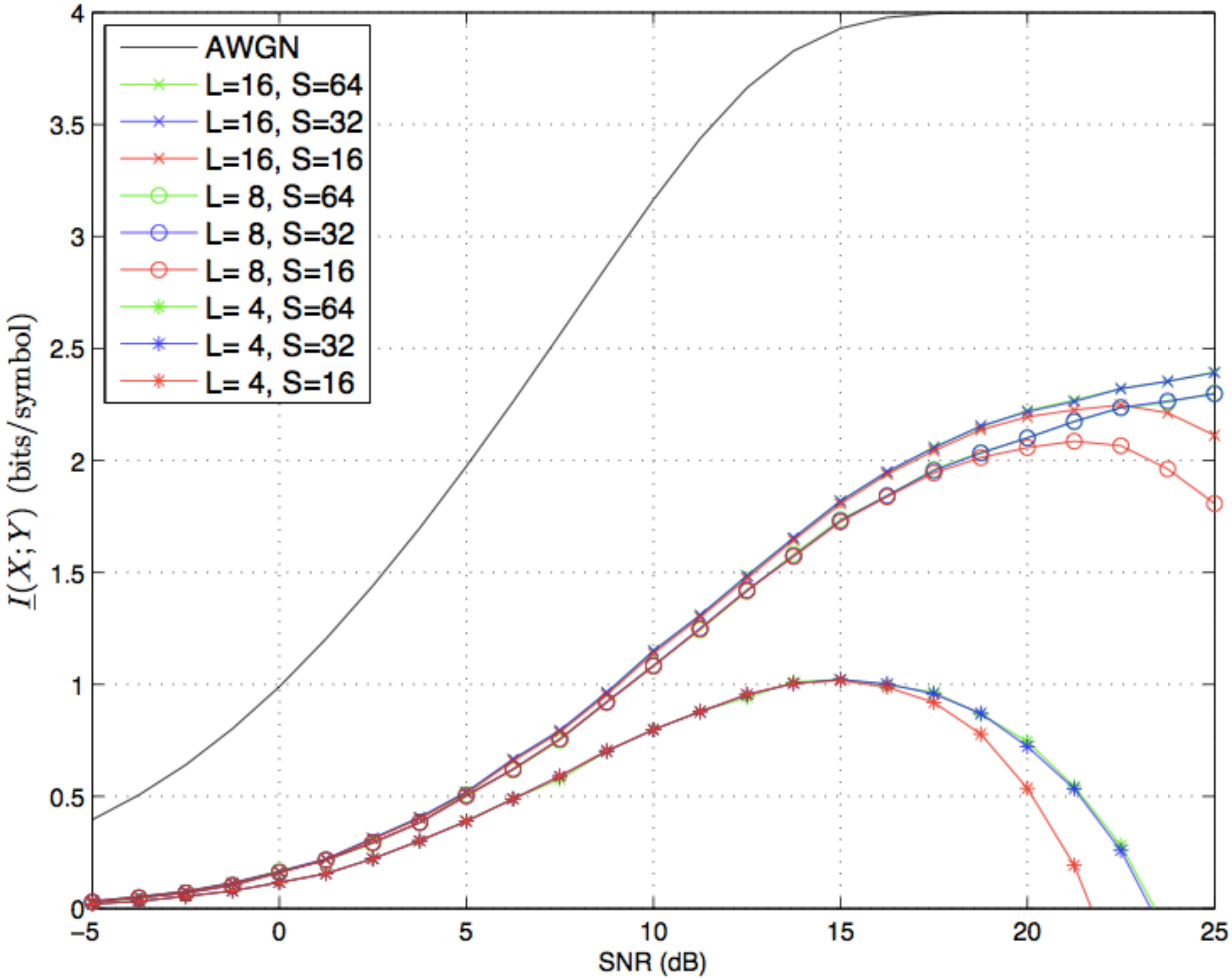
- Rectangular and cosine-squared pulse shapes
- QPSK, 16-PSK, and 16-QAM
- Excessively large* linewidth: $\beta\, T_{symb}=0.125$
  Large* linewidth: $\beta\, T_{symb}=0.0125$
- Large linewidths are chosen to simplify simulations.
- Observations:
  - the same qualitative behavior occurs for any linewidths and high SNR
  - the information rates are quite good even for such linewidths

* Critique: both much larger than for many popular oscillators, but new applications (wireless & optical MIMO, machine-to-machine) are emerging where cheap oscillators are needed

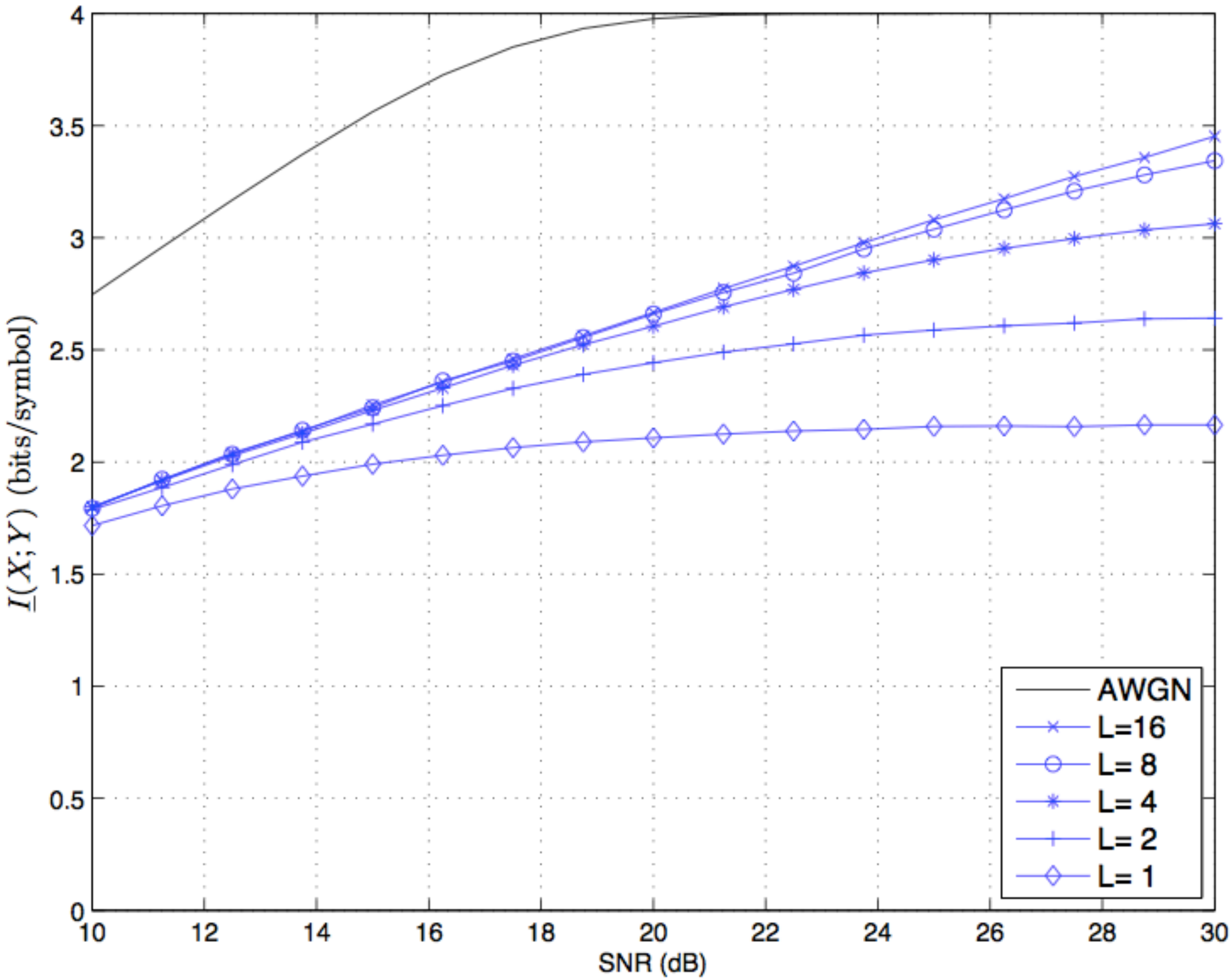
- Lower bounds for 16-QAM
- $\beta\, T_{symb}$=0.125
- Rectangular pulse shape



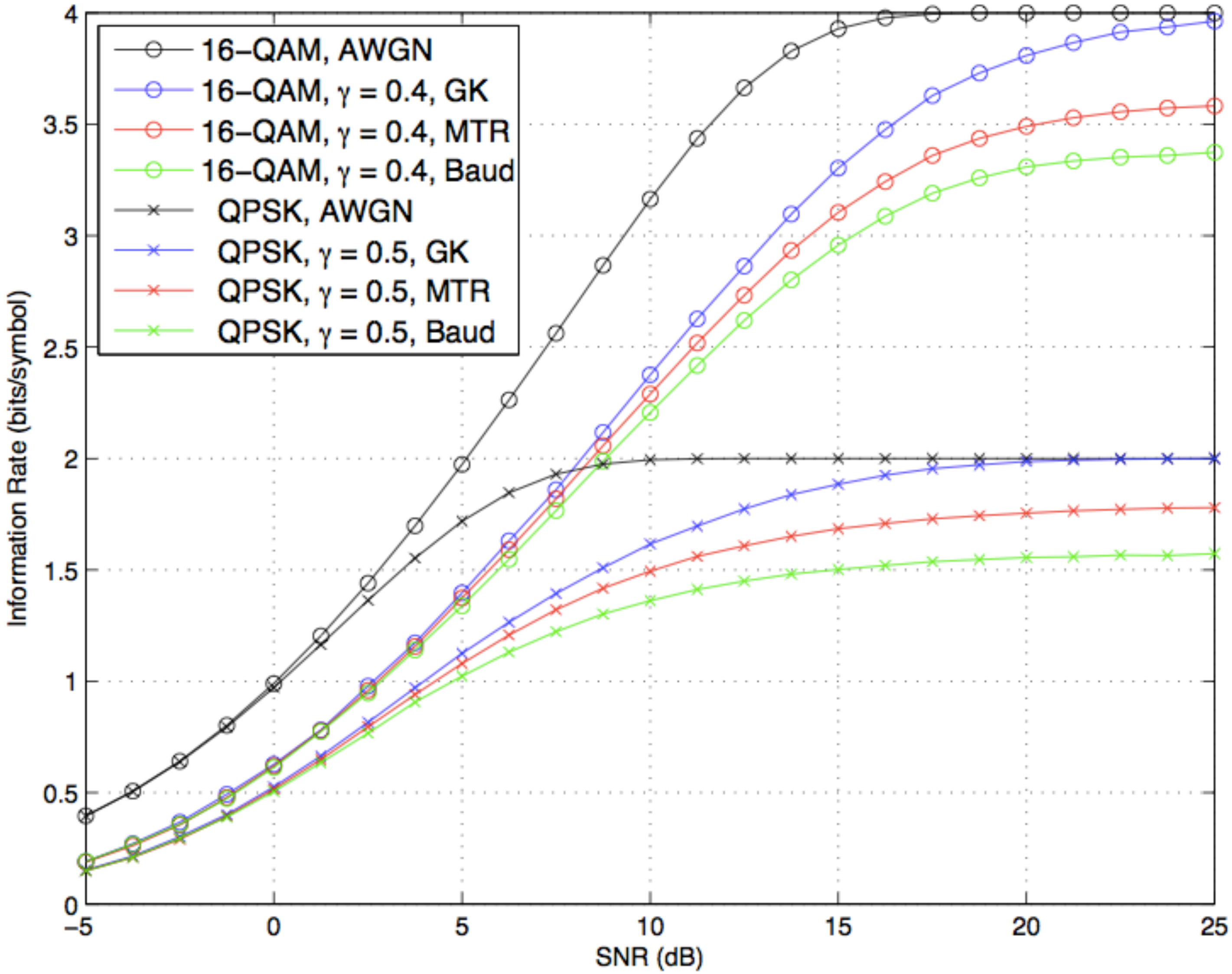
- Lower bounds for 16-QAM
- $\beta\, T_{symb}$=0.125
- Cosine-squared pulse shape



- Lower bounds for 16-PSK
- $\beta\, T_{symb}$=0.0125
- Rectangular pulse shape
- S=64

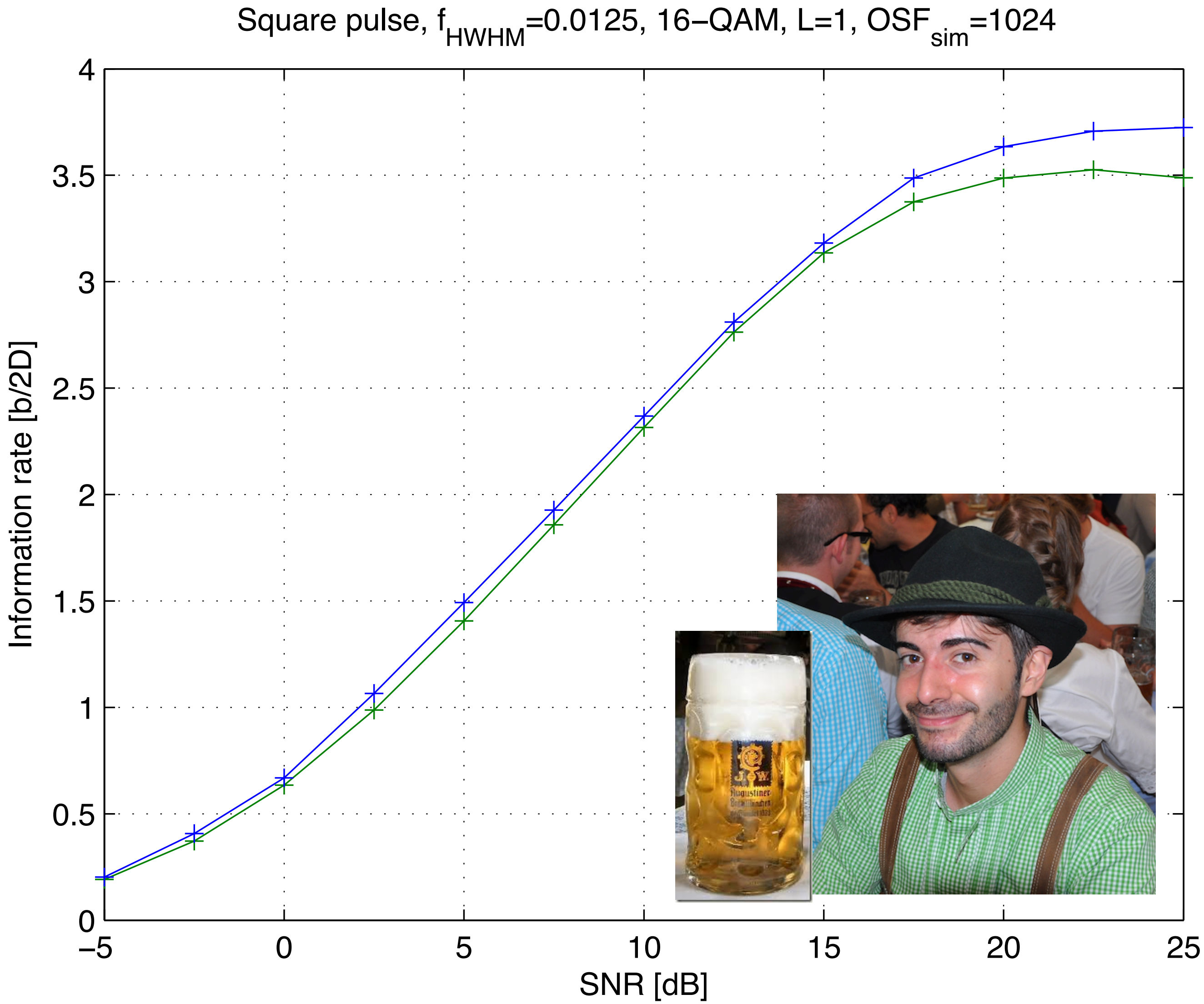


- Lower bounds for rectangular pulse
- $\gamma^2 = 2\pi \beta T_{symb}$
- Baud-rate model with L=1, F_k=1
- MTR model*: Discrete-time OS, matched filter, L=16, F_k=1
- NB: L=16 achieves $\log_2(M)$ bits/symbol; M = modulation size
- S=128; except S=64 for 16-QAM (due to complexity)



* Martalo-Tripoli-Raheli (ITA 2013)

- Upper and lower bounds for rectangular pulse*
- f_{HWHM}=0.0125 means γ =0.4
- Baud-rate sampling
- Baud rate sampling cannot achieve 4 bits per symbol; so we need OS



* Curves courtesy of Luca Barletta

Summary

- OS is important at high SNR for any linewidth. The required OS rate depends on the (1) linewidth, (2) SNR, (3) pulse shape
- For Wiener phase noise: the required L seems to grow as the third root of the SNR (can we do better? relate to $S_U(f) \sim 1/f^2$?)
- Many papers use an approximate discrete time model even at high SNR. Exercise caution: accurate models have amplitude variations also, <u>especially</u> at high SNR
- Lots of basic, fun, open problems*. For example, find (a good bound on) the differential entropy of

$$F_1 = \frac{1}{\Delta} \int_0^{\Delta} e^{j\Theta(t)}\, dt$$

* Several consequences for OFDM !

Extra Slides

Research Activities at LNT

Codes for Iterative Decoding

Cooperative Communications

Algebraic Subspace Coding

Information & Coding & Communications Theory & Signal Processing

Interference Management

Network Inform. Theory

Network Coding

Fiber Optic Capacity

Security Coding

Phase Noise Channels