

## Nonnegativity of $\mu^*$ for $X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4$

1. It can be proved that  $\mu^*$  vanishes on the 5 atoms shown in the information diagram.
2. Suppress these atoms by setting them to  $\emptyset$  to obtain the information diagram below.
3. From this information diagram, it can readily be checked that the values of  $\mu^*$  on the remaining

$$(2^4 - 1) - 5 = 10$$

nonempty atoms are equal to

$$\begin{aligned} &H(X_1|X_2, X_3, X_4), I(X_1; X_2|X_3, X_4) \\ &I(X_1; X_3|X_4), I(X_1; X_4) \\ &H(X_2|X_1, X_3, X_4), I(X_2; X_3|X_1; X_4) \\ &I(X_2; X_4|X_1), H(X_3|X_1, X_2, X_4) \\ &I(X_3; X_4|X_1, X_2), H(X_4|X_1, X_2, X_3). \end{aligned}$$

3. There are all Shannon's information measures which are always nonnegative. Therefore,  $\mu^*$  is a measure.

**Exercise:** Identify these 10 atoms in the information diagram at the bottom.