1. Consider a Type I atom A such that

$$\mathcal{N}_n \backslash U_A = \{l, l+1, \cdots, u-1, u\},$$

where $1 \leq l \leq u \leq n$.

2. Define the set

$$W = \{l+1, \cdots, u-1\}.$$

3. Then

$$\begin{aligned}
&I(X_l; X_u | X_{U_A}) \\
&\quad = \mu^*(\tilde{X}_l \cap \tilde{X}_u - \tilde{X}_{U_A}) \\
&\quad = \mu^*\left(\bigcup_{S \subseteq W}\left(\tilde{X}_l \cap \left(\bigcap_{t \in S} \tilde{X}_t\right) \cap \tilde{X}_u - \tilde{X}_{U_{A \cup (W-S)}}\right)\right) \\
&\quad = \sum_{S \subseteq W} \mu^*\left(\tilde{X}_l \cap \left(\bigcap_{t \in S} \tilde{X}_t\right) \cap \tilde{X}_u - \tilde{X}_{U_{A \cup (W-S)}}\right).
\end{aligned}$$

4. In the above summation, except for the atom corresponding to $S = W$, namely

$$(\tilde{X}_l \cap \tilde{X}_{l+1} \cap \cdots \cap \tilde{X}_u - \tilde{X}_{U_A}),$$

all the atoms are Type II atoms.

5. Therefore,

$$I(X_l; X_u | X_{U_A}) = \mu^*(\tilde{X}_l \cap \tilde{X}_{l+1} \cap \cdots \cap \tilde{X}_u - \tilde{X}_{U_A}).$$

6. Hence,

$$\begin{aligned}
\mu^*(A) &= \mu^*(\tilde{X}_l \cap \tilde{X}_{l+1} \cap \cdots \cap \tilde{X}_u - \tilde{X}_{U_A}) \\
&= I(X_l; X_u | X_{U_A}) \\
&\geq 0.
\end{aligned}$$