4. For each $J \subseteq \cup_i Q_i$, let $J_i = J \cap Q_i$. Then

$$J = \bigcup_{i=1}^{m} J_i.$$

5. Since $X_{Q_1}, X_{Q_2}, \cdots, X_{Q_m}$ are mutually independent conditioning on X_T, so are $X_{J_1}, X_{J_2}, \cdots, X_{J_m}$.

6. Following (1), we have

$$\begin{aligned} \mu^*(A) &= \sum_{J \subseteq \cup_i Q_i} (-1)^{|J|+1} \sum_{i=1}^{m} H\left(X_{J_i} | X_T\right) \\ &= \sum_{i=1}^{m} \sum_{J_i \subseteq Q_i} \alpha(J_i) H\left(X_{J_i} | X_T\right), \end{aligned}$$

where $\alpha(J_i)$ is the coefficient of $H\left(X_{J_i} | X_T\right)$ in the summation.

7. By simple counting, we obtain that

$$\begin{aligned} \alpha(J_i) &= \sum_{L \subseteq \cup_{j \neq i} Q_j} (-1)^{|J_i|+|L|+1} \\ &= (-1)^{|J_i|+1} \sum_{L \subseteq \cup_{j \neq i} Q_j} (-1)^{|L|} \\ &= 0, \end{aligned}$$

because

$$\sum_{L \subseteq \cup_{j \neq i} Q_j} (-1)^{|L|} = 0$$

by the binomial formula. Hence $\mu^*(A) = 0$.

8. Now consider a general atom in $Im(K)$:

$$A = \left(\bigcap_{i=1}^{k} \bigcap_{j \in W_i} \tilde{X}_j \right) - \tilde{X}_{T \cup (\cup_{i=1}^{k} (Q_i - W_i))}$$

where $W_i \subseteq Q_i$ and there exist at least two i such that $W_i \neq \emptyset$.

9. By Theorem 12.5, $X_{W_1}, X_{W_2}, \cdots, X_{W_m}$ are mutually independent conditioning on X_T, $X_{Q_1 - W_1}$, $X_{Q_2 - W_2}, \cdots, X_{Q_m - W_m}$, i.e., the FCMI

$$\left(T \cup (\cup_{i=1}^{k} (Q_i - W_i)), W_1, W_2, \cdots, W_m\right)$$

holds. Then repeat the above argument to see that $\mu^*(A) = 0$.