On Spectral Efficiency of First Order Soliton-Based Optical Communications

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6th Van Der Meulen Seminar, Eindhoven

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Outline

Agenda

Basis of this Presentation

- Doctoral work of Alexander Span, Institute of Telecommunications, University of Stuttgart
- Material taken from paper
  - A. Span, S. ten Brink, "On spectral efficiency of first order soliton-based optical communications," IEEE Biennial Symposium on Communications, Canada, June 2016.
- contact the authors
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- check out related webdemo "Soliton Impulses" at http://www.inue.uni-stuttgart.de/lehre/demo.html

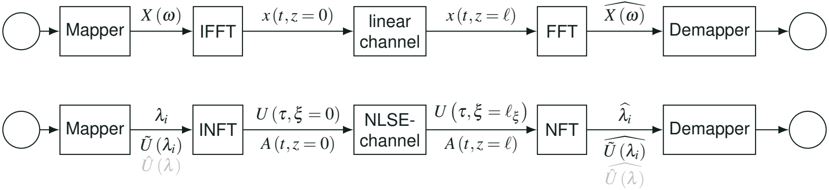
Brief History of Optical Communications

- Until about 2000:
  - Intensity-based modulation (on/off keying)
  - Data rates: a few Gb/s
- Differential, auto-coherent detection, until mid 2000‘s
  - 10s of Gb/s
- Coherent modulation/detection, mid 2000‘s-today
  - QAM
  - Exploits polarization multiplex
  - Dispersion (chromatic, polarization) „contained“
- Next frontier: mitigate non-linearities
  - for higher rate, farther reach

Modulation for Linear/Non-linear Channels

- Linear medium (e.g. wireless channel)
  - Sinusoidal Eigenfunctions, i.e., exp(j. . .)
  - Allows multicarrier modulations (e.g., OFDM)
  - Use Fourier Transform for modulation/demodulation (iFFT, FFT)
- Non-linear medium (e.g. optical fiber)
  - Eigenfunctions? Well, depends on the type of non-linearity. . .
  - For optical medium described by NLSE: Solitons!
  - „Multi-soliton modulation“ corresponds to OFDM in linear case
  - Use Non-linear Fourier Transform (NLFT)

Communication Link



- top: conventional multicarrier OFDM/FFT with sinusoidals $\exp(j\omega_c t)$ as information carriers
- bottom: NLFT-based communication with solitons as carriers

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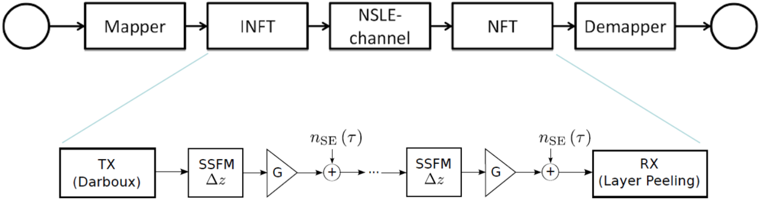
Non-linear Channel Model



- SSFM: non-linear channel is quite complicated to simulate... there is no short-cut

Non-linear Channel Model

- Non-linear Schrödinger Equation

$$\frac{\partial A}{\partial z} - j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + j\gamma |A|^2 A = 0$$

- Split-Step Fourier (transform) Method SSFM

$$A(t,z_0+\Delta z) = \mathscr{F}^{-1}\left\{ \underbrace{\mathscr{F}\left\{ \underbrace{\overbrace{A(t,z_0)\exp\left(-j\gamma|A(t,z_0)|^2\Delta z\right)}^{\text{Kerr effect (time domain)}}\exp\left(-\alpha\Delta z\right)}_{\text{+attenuation (time domain)}} \right\}\exp\left(-j\frac{\beta_2\omega^2}{2}\Delta z\right)}_{\text{+chromatic dispersion (frequency domain)}} \right\}$$

- attenuation coefficient $\alpha = 0.2\mathrm{dB/km}$
- dispersion coefficient $\beta_2 = -2.03 \cdot 10^{-26}\,\mathrm{s^2/m}$
- nonlinearity coefficient $\gamma = 1.62 \cdot 10^{-26}\,\mathrm{m/V^2}$

Some Effects on Optical Fiber

- What‘s linear on the optical channel?
  - Refraction index depends on wavelength
  - Leads to chromatic dispersion
    - All-pass, phase quadratic in frequency
  - Can be compensated using linear filters
  - best simulated in frequency-domain
- What‘s non-linear?
  - Kerr-effect: refraction index depends on optical power
  - Compensation. . . digital back-propagation. . . difficult. . .
  - best simulated in time-domain
- Higher data rate or farther reach require higher launch powers
  - then, Kerr-nonlinearity can no longer be neglected
- Idea: don‘t fight it, but „exploit“ it
  - Mitigate non-linearities by solitonic communications

Evolution of a Topic (1/2)

Optical Communications Using the Nonlinear Fourier Transform

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    - [8] M. I. Yousefi, F. R. Kschischang, Nonlinear Fourier Transform, Part III: Spectrum Modulation, 2014

Evolution of a Topic (2/2)

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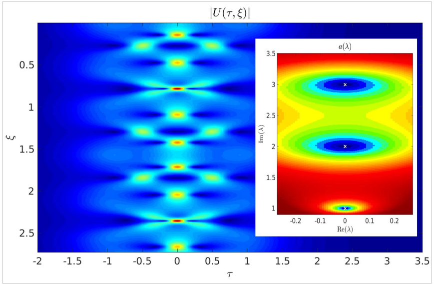
- note: not a complete list...
- our recent contribution this talk is based on: A. Span, S. ten Brink, "On spectral efficiency of first order soliton-based optical communications," IEEE Biennial Symposium on Communications, Canada, June 2016

Agenda

Evolution Equations

NFT Evolution Equations (Nonlinear Convolution)

$$\lambda_i(\xi) = \lambda_i(\xi = 0)$$
$$\widetilde{U}(\lambda_i, \xi) = \widetilde{U}(\lambda_i, \xi = 0)\exp\left(-4j\lambda_i^2\xi\right)$$
$$\widehat{U}(\lambda, \xi) = \widehat{U}(\lambda, \xi = 0)\exp\left(-4j\lambda^2\xi\right)$$



- linear evolution equations
- simple description of signal propagation in nonlinear frequency domain

3rd Order Soliton Pulse, Time-Domain

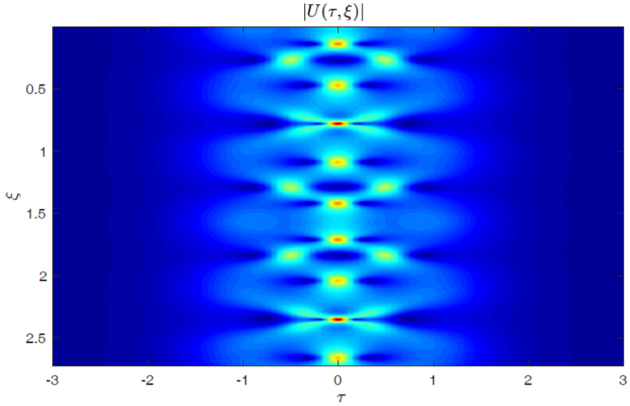


Figure 3.38: Propagation of third order soliton in time domain with eigenvalues $\lambda_1 = 1j$, $\lambda_2 = 2j$, $\lambda_3 = 3j$

- higher order solitons show typical “breathing” effect

3rd Order Soliton Pulse, (lin.) Frequency-Domain

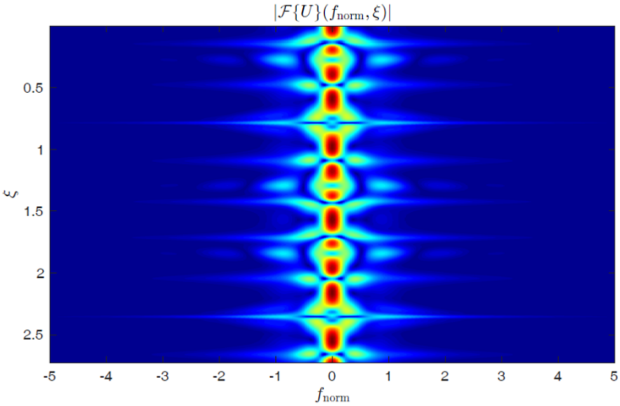


Figure 3.39: Propagation of third order soliton in frequency domain with eigenvalues $\lambda_1 = 1j, \lambda_2 = 2j, \lambda_3 = 3j$

- breathing effect in both time- and (lin.) frequency-domain

How to Define „Bandwidth“, „Spectral Efficiency“?

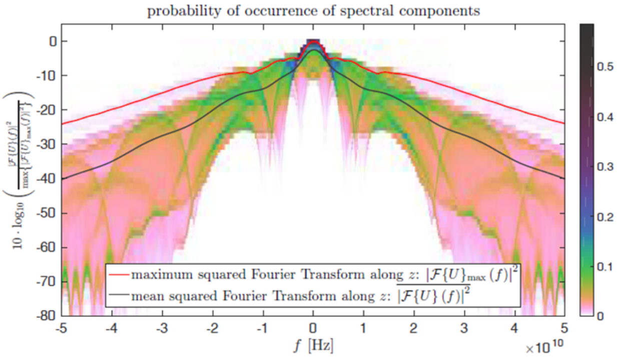
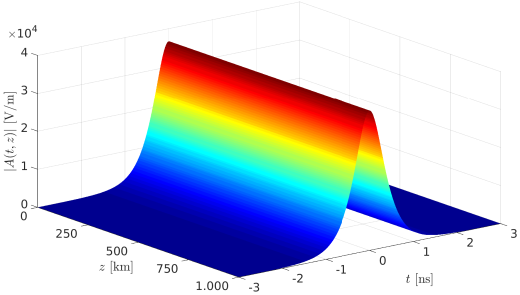


Figure 3.40: Probability of occurrence of spectral components for third order soltion with $\lambda_1 = 1j$, $\lambda_2 = 2j$, $\lambda_3 = 3j$ for a propagation distance of $z = 1000\,\text{km}$

- bandlimited/filtered channels (e.g. WDMA)... how to define "bandwidth"

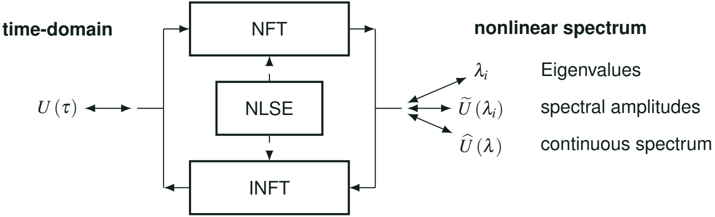
Focus on First Order Soliton Modulation

- First Order: only one eigenvalue ⇒ analytical solution
- (continuous spectrum is set to zero)
- impulse fully determined by eigenvalue and fiber parameters



- dispersion and nonlinearity balance each other
- first order soliton keeps its shape during propagation (no breathing in time/frequency)

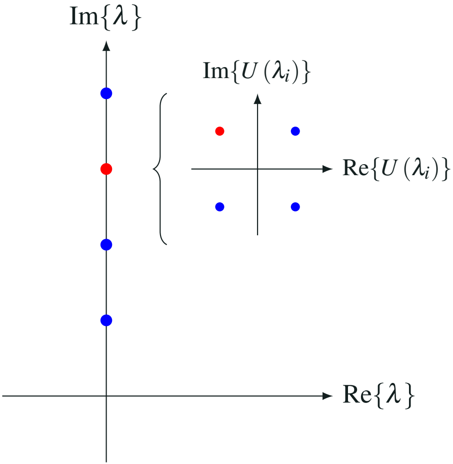
Degrees of Freedom



- discrete spectrum (solitonic part)
  - described by eigenvalues (EVs), spectral amplitudes
- continuous spectrum (dispersive part)
  - converges to ordinary Fourier spectrum for small amplitudes
- EVs, their spectral amplitudes, and cont. spectrum are independent degrees of freedom: use for communication!

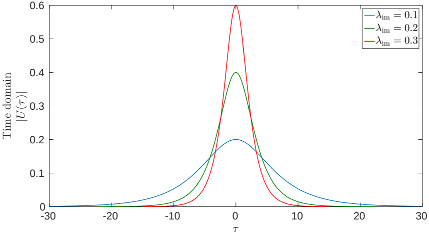
Information Encoding - Degrees of freedom

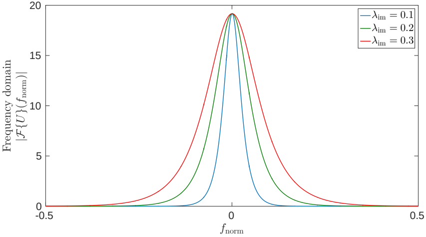
- considered in this work
  - 1-out-of-N *arbitrary* eigenvalue constellation; fixed spectral amplitude
  - or: 1-out-of-M *arbitrary* spectral amplitude constellation; fixed eigenvalue



Which constellation for good spectral efficiency?

Eigenvalue Imaginary Part - Time/BW scaling

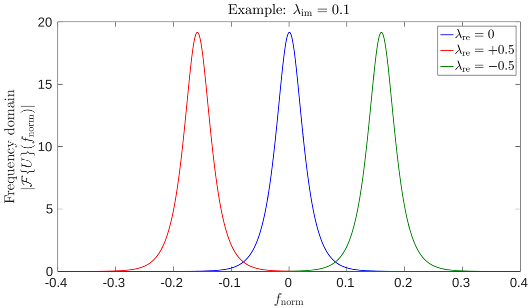




- duration $T \sim \frac{1}{\lambda_{\text{im}}}$
- bandwidth $B \sim \lambda_{\text{im}}$

Result: Time-Bandwidth-Product of constellation follows $TB \sim \frac{\lambda_{\text{im,max}}}{\lambda_{\text{im,min}}}$

Eigenvalue Real Part - Frequency Shift

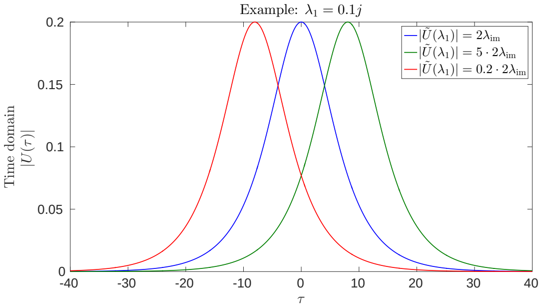


- frequency shift $f_{\text{shift}} \sim \lambda_{\text{re}}$
- $\Rightarrow$ temporal shift due to change in group velocity $T_{\text{shift}} \sim \lambda_{\text{re}}$

Result: TB-Product of constellation follows

$$TB \sim \left(a\frac{1}{\lambda_{\text{im}}} + b\lambda_{\text{re,max}}\right)(c\lambda_{\text{im}} + d\lambda_{\text{re,max}})$$

Spectral Amplitude - Temporal Shift



- temporal shift $T_{\text{shift}} \sim \ln\left(\frac{|\tilde{U}|}{2\lambda_{\text{im}}}\right)$ → increasing time slot

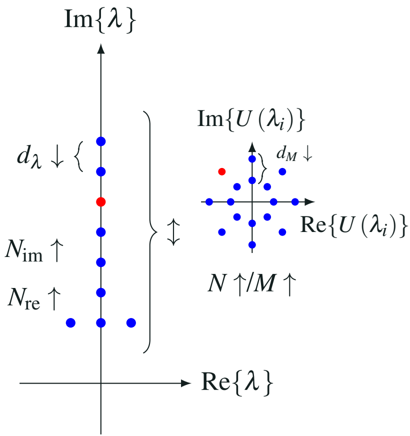
Result: Time-Bandwidth-Product of constellation follows

$$TB \sim \ln\left(\frac{|\tilde{U}_{\max}|}{|\tilde{U}_{\min}|}\right)$$

Trading off Parameters for Increasing SE

Requirements: $T\downarrow B\downarrow I\uparrow$

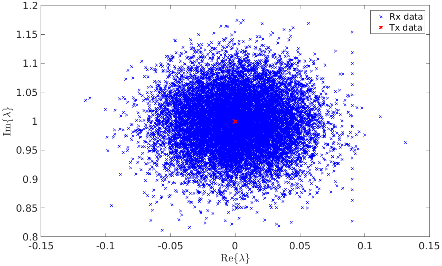
- small temporal impulse spacing $\delta_T\uparrow$
- small ratio of $\frac{\lambda_{\text{im,max}}}{\lambda_{\text{im,min}}}\downarrow$
- small difference of $\lambda_{\text{re,max}}-\lambda_{\text{re,min}}$
- large number of eigenvalue constellation points $N_{\text{re}}\uparrow/N_{\text{im}}\uparrow$
- $\Rightarrow$small eigenvalue spacing $d_\lambda\downarrow$
- small ratio of $\frac{|\widetilde{U}_{\max}|}{|\widetilde{U}_{\min}|}\downarrow$
- large number of spectral amplitude constellation points $N\uparrow/M\uparrow$
- $\Rightarrow$small spacing $d_M\downarrow$

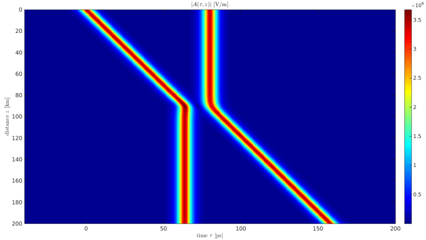


Limiting Factors

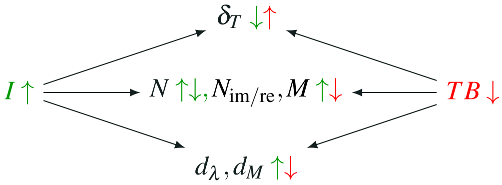
Problems for Detection

- noise induced eigenvalue / spectral amplitude deviation
- eigenvalue deviation due to truncation and impulse interaction
- $\Rightarrow N_{\text{im}} \uparrow \longleftrightarrow \frac{\lambda_{\text{im,max}}}{\lambda_{\text{im,min}}} \downarrow$
- $\Rightarrow N_{\text{re}} \uparrow \longleftrightarrow \lambda_{\text{re,max}} - \lambda_{\text{re,min}} \downarrow$
- $\lambda_{\text{im,max}}$ limited by maximum sampling frequency ($\rightarrow$ also power limitation)
- *noise and interaction are signal dependent*





Parameter Optimization for high SE



... $f(\lambda_{\mathrm{im}})$

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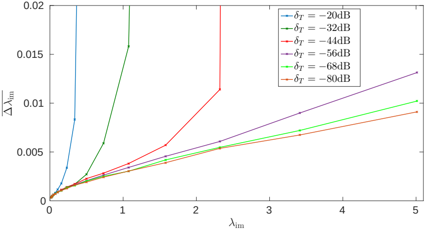
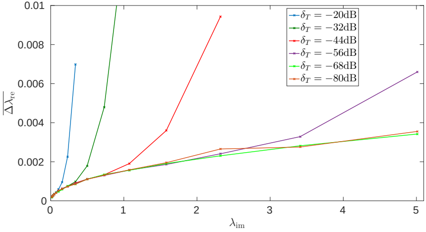
Post-Detection Eigenvalue Deviation

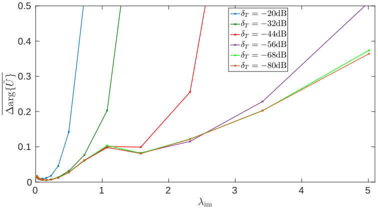




- imaginary part eigenvalue deviation
- real part eigenvalue deviation

- noise power and interaction increase for larger λ_{im}
- nonlinear impulse interaction increases for smaller impulse spacing
- ideal distributed Raman ampl., SSMF parameters, $1000\mathrm{km}$

Post-Detection Spectral Amplitude Deviation



- spectral phase deviation

- noise power and interaction increase for larger λ_{im}
- deviation in eigenvalue λ increases deviation of spectral amplitude $\widetilde{U}$
- ideal distributed Raman ampl., SSMF parameters, 1000km

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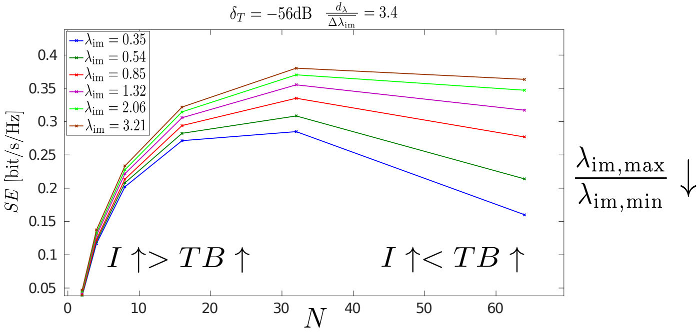
Imaginary Part Constellation

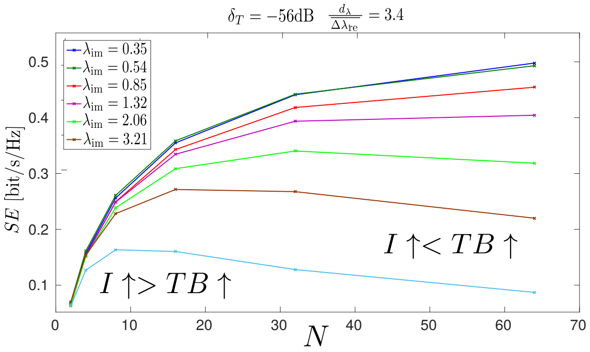
Example:



- $N \uparrow$: I grows faster than TB / $N \uparrow\uparrow$: TB grows faster than I
- $\lambda_{\text{im}} \uparrow$: TB decreases as $\frac{\lambda_{\text{im,max}}}{\lambda_{\text{im,min}}} \downarrow$ / $\lambda_{\text{im}} \uparrow\uparrow$: TB increases as eigenvalue spacing needs to become larger (deviations)

Real Part Constellation

Example:



- $N \uparrow$: I grows faster than TB / $N \uparrow\uparrow$: TB grows faster than I
- $\lambda_{\text{im}} \uparrow$: TB increases as eigenvalue spacing needs to become larger (deviations)

Agenda

Summary

- NLSE propagation can easily be described in nonlinear spectral domain
  $\rightarrow$ encode information in nonlinear spectrum
- First order soliton: stable impulse determined by eigenvalue

- some approximated simulation results for SE
- SE, so far, quite low (typ. 0.5..1.5b/s/Hz); lots of work to be done!

- How to do better?
  - combination of real/imaginary part and spectral amplitude
  - use higher order solitons (multiple eigenvalues in parallel)
  - use continuous spectrum

Thank you for your attention!

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